Dynamic Model of an Electro-hydraulic Three Point Hitch

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***Abstract*— This paper proposes a dynamic model of an electro-hydraulic three point hitch for farm tractors. The modeling technique is based on the Hamiltonian system framework and on power-port interaction between subsystems. The model allows fast and precise simulations and therefore can be used for the development and the validation of control strategies by simulations and hardware in the loop experiments. The proposed model has been validated comparing the simulation results with experimental measurements.**

I. INTRODUCTION

A three point hitch for farm tractors is a fundamental device for the agricultural works. It's typical framework is shown in Fig. 1, it is composed by a set of levers that connect the agricultural implement (or tool) to the tractor chassis. The levers are operated by a lift cylinder controlled by two electro-valves as shown in Fig. 2.

Two kind of control targets are usually considered: position control and draft control. The position controller must keep the position of the implement to a desired value facing the disturbances on the load. The draft controller aims to keep constant the draft on the hitch levers, it requires a draft sensor but it easily prevents the farm tractor from stalling in case of a strong draft increase. Combined position/draft controllers are usually implemented.

As described in [1] the performance of an hitch controller is usually evaluated by hours of experimental measurements and field tests. This procedure becomes very expensive when it is carried out for the development of a new controller: trial and error experiments requires hours of tests and the parameters tuning is often based more on expertize than on numerical evaluations.

The aim of this work is the identification of a dynamic model of an electro-hydraulic three point hitch for the validation of control strategies by simulations and hardware in the loop (HIL) experiments.

The presence of three energetic domains (electrical, hydraulic and mechanical) and the kinematics of the levers make this modeling problem challenging. The modeling approach followed in the paper is based on power interaction between subsystems as described in [2] and [3]. One of the key modeling issues is due to the hitch kinematics. This issue is faced by modeling the hitch as an Hamiltonian system, please refer to [4] and [5] for deeper details. The proposed hitch submodel takes into account also the friction phenomena due to static and Coulomb friction.

Faster simulations are essential to allow HIL experiments. The static and Coulomb friction are properly simulated to avoid switching phenomena that increases abruptly the computational effort and the simulation time. To get faster simulations, the complex mathematical description of the hitch dynamics is properly simplified maintaining the physical coherence of the model (i.e. no false energy generation/dissipation processes).

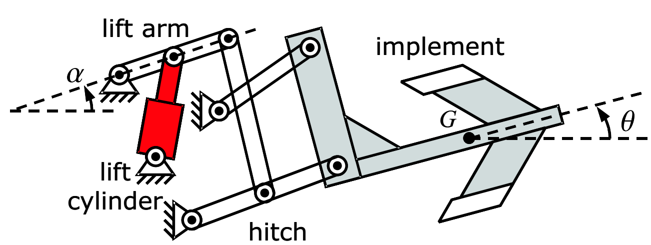

Fig. 1. Three point hitch for a farm tractor.

The paper is organized as follows: Section II gives a brief introduction on Hamiltonian systems that are the base of the model presented in Section III. This Section present also the algorithm for the fast simulation of the static and Coulomb friction and the model simplification. Despite this simplification the proposed model is able to reproduce the main dynamic behavior of the system as shown in Section IV. Finally some conclusions are drawn.

II. A BRIEF INTRODUCTION ON HAMILTONIAN SYSTEMS

The hamiltonian framework is a powerful means to model dynamic systems. A brief recall of the definitions given in [4] and in [5] is given herein for reader convenience.

The standard *Euler-Lagrange equations* are given as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial L}{\partial q}(q,\dot{q}) = \tau \qquad (1)$$

where $q = (q_1, \ldots, q_n)^T$ are generalized configuration coordinates for the system with n degrees of freedom,

$\dot{q} = (\dot{q}_1, \ldots, \dot{q}_n)^T$ are the generalized velocities, $\tau = (\tau_1, \ldots, \tau_n)^T$ is the vector of the generalized forces acting on the system, the *Lagrangian* $L(q, \dot{q})$ equals the difference between the kinetic energy $K(q, \dot{q})$ and the potential energy $V(q)$:

$$L(q, \dot{q}) = K(q, \dot{q}) - V(q)$$

The partial derivatives $\frac{\partial L}{\partial \dot{q}}$ and $\frac{\partial L}{\partial q}$ are column vector. The Lagrangian function $L(q, \dot{q})$ in standard mechanical systems is of the form:

$$L(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q)\, \dot{q} - V(q) \tag{2}$$

where the $n \times n$ inertia (generalized mass) matrix $M(q)$ is symmetric and positive definite for all q.

The vector of generalized *momenta* $p = (p_1, \ldots, p_n)^T$ is defined for any Lagrangian as:

$$p = \frac{\partial L(q, \dot{q})}{\partial \dot{q}}$$

For a standard mechanical systems with Lagrangian (2) the generalized momenta are of the form:

$$p = M(q)\, \dot{q} \tag{3}$$

By defining the state vector $(q_1, \ldots, q_n, p_1, \ldots, p_n)^T$ the n second-order equations (1) transforms into a $2n$ first-order equations called *Hamiltonian equations* of motion:

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + \tau \end{aligned} \tag{4}$$

where the *Hamiltonian* $H(q, p)$ is the total energy of the system:

$$H(q, p) = K(q, p) + V(q)$$

System (4) is an example of a *Hamiltonian system* which more generally is given in the following form:

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + B(q) u \\ y &= B^T(q) \frac{\partial H}{\partial p}(q, p) = B^T(q) \dot{q} \end{aligned} \tag{5}$$

where $B(q) \in \mathbf{R}^{n \times m}$ is the input force matrix, $B(q)\, u$ denotes the generalized forces resulting from the control inputs $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^m$ are the outputs. The power flowing into the system is:

$$\frac{dH}{dt}(q(t), p(t)) = u^T(t)\, y(t)$$

therefore the pair (u, y) represents a *power-port* between the hamiltonian system and the external world.

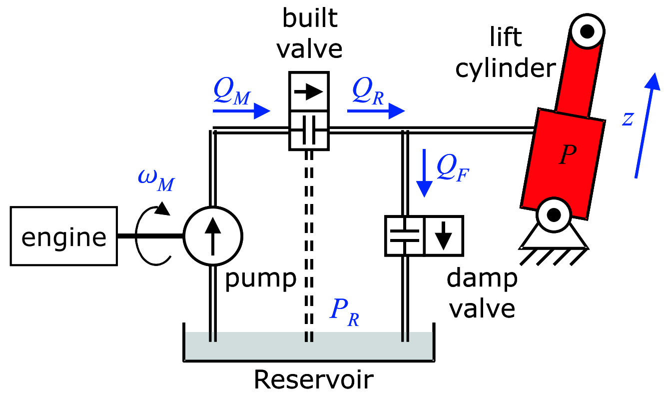

Fig. 2. Schematic of the standard electro-hydraulic components of a three point hitch.

III. THREE POINT HITCH MODEL

This Section presents the two main submodel of the proposed model: the hydraulic actuators model (in Subsection A) and the hitch levering dynamic model (in Subsection B). Subsection C introduces the simulation procedure that deals with the friction phenomena. Finally, subsection D describes a model simplification that allows both to speed up the simulations and to maintain the physical coherence of the model.

A. Hydraulic Actuator dynamic model

The hydraulic actuator is composed by a hydraulic pump, a lift cylinder and two electro-valves as shown in Fig. 2.

The pump is operated by the engine and provides a oil flow Q_M proportional to the engine revolution speed ω_M: $Q_M = K_M\, \omega_M$.

The built valve is a *flow control valve* and the oil flow Q_R toward the lift cylinder is modeled as:

$$\begin{aligned} Q_R &= \min\{Q_M\,,\ f_R(u_R)\} \\ \dot{u}_R &= \frac{1}{\tau_R}(i_R - u_R) \end{aligned} \tag{6}$$

where i_R is the control current, u_R is a state variable for the valve dynamics and $f_R(u_R)$ is a nonlinear map from the state variable u_R and the output oil flow. The time constant τ_R models the main dynamic behaviour of the valve. Equations (6) give a “logic” description of the valve behaviour, this kind of modeling is suitable for the hitch model since the internal valve phenomena are much faster than the other dynamics in the system and the valve flow results almost independent from the pressure P within the lift cylinder.

The damp valve is a controlled discharge orifice, the discharge flow Q_F is:

$$\begin{aligned} Q_F &= C_F(u_F) \sqrt{|P - P_R|}\, \mathrm{sign}(P - P_R) \\ \dot{u}_F &= \frac{1}{\tau_F}(i_F - u_F) \end{aligned} \tag{7}$$

the first equation is the well known relation that gives the oil flow through an orifice as a function of the pressure drop across the orifice $P - P_R$. The oil density is considered to be constant. u_F is a state variable for the valve dynamics and $C_F(u_F)$ is the function that describes the *effective area* of the orifice as a function of the internal state variable u_F. i_F is the valve control current.

Let z be the linear displacement of the lift cylinder and let α denote the angular position of the lift arm as shown in Fig. 1. The nonlinear relation between z and α is:

$$z = f_Z(\alpha) \qquad \Rightarrow \qquad \dot{z} = \frac{d\, f_Z(\alpha)}{d\,\alpha}\,\dot{\alpha} \tag{8}$$

The derivative of $f_Z(\alpha)$ allows to describe the power-port connection between the cylinder and the hitch levers:

$$\begin{aligned} \tau_p &= A_z\, \frac{d\, f_Z(\alpha)}{d\,\alpha}\, P \\ Q_z &= A_z\, \dot{z} = A_z\, \frac{d\, f_Z(\alpha)}{d\,\alpha}\,\dot{\alpha} \end{aligned} \tag{9}$$

where τ_p is the torque on the lift arm due to the pressure P, A_z is the lift cylinder active area and Q_z is the oil flow through the lift cylinder due to the lift arm rotation.

The pressure dynamics is finally modeled by an hydraulic capacitance C_z:

$$C_z\, \dot{P} = Q_R - Q_F - Q_z \tag{10}$$

B. Hitch levering dynamic model

The dynamic model of the hitch levering is obtained by following the port-hamiltonian framework. Let (x, y) denote the position of the center of gravity G of the implement with respect to an inertial frame, let θ be the angular position of the implement as shown in Fig. 1. The mass and the inertia (respect to the rotation θ around G) of the implement are denoted with L and J respectively. The mass of the levering arms can be neglected with respect to the mass of the implement.

The levering system has only one degree of freedom, therefore it is possible to compute the position (x, y, θ) of G as a function of the angular position α of the lift arm:

$$x = f_x(\alpha) \qquad y = f_y(\alpha) \qquad \theta = f_\theta(\alpha) \tag{11}$$

The Hamiltonian H of the hitch levering is the kinetic energy of the implement (the potential energy due to the gravity is considered later):

$$H = \frac{1}{2} L\, \dot{x}^2 + \frac{1}{2} L\, \dot{y}^2 + \frac{1}{2} J\, \dot{\theta}^2$$

computing the time derivatives using (11):

$$H = \frac{1}{2}\, M(\alpha)\, \dot{\alpha}^2 \tag{12}$$

where

$$M(\alpha) = L \left(\frac{d\, f_x(\alpha)}{d\,\alpha} \right)^2 + L \left(\frac{d\, f_y(\alpha)}{d\,\alpha} \right)^2 + J \left(\frac{d\, f_\theta(\alpha)}{d\,\alpha} \right)^2 \tag{13}$$

From (3), the momenta p_α is then given by:

$$p_\alpha = M(\alpha)\, \dot{\alpha} \tag{14}$$

and the corresponding Hamiltonian is finally:

$$H(\alpha, p_\alpha) = \frac{1}{2} \frac{p_\alpha^2}{M(\alpha)} \tag{15}$$

From a power-port viewpoint, the hitch levering has two power-ports: the first, toward the lift arm, is described by the pair $(\tau_\alpha, \dot{\alpha})$, the second describes the interaction between the center of gravity of the implement and the environment and it is given by $([F_x, F_y, \tau_\theta]^T, [\dot{x}, \dot{y}, \dot{\theta}]^T)$. This second power-port takes into account also the gravity.

Given the Hamiltonian (15) and the two power-ports described above, the equations of motion of the hitch levering result:

$$\begin{aligned} \dot{\alpha} &= \frac{\partial H}{\partial p_\alpha}(\alpha, p_\alpha) = \frac{p_\alpha}{M(\alpha)} \\ \dot{p}_\alpha &= -\frac{\partial H}{\partial \alpha}(\alpha, p_\alpha) + \tau_\alpha + \frac{d\, f_x}{d\,\alpha} F_x + \frac{d\, f_y}{d\,\alpha} F_y + \frac{d\, f_\theta}{d\,\alpha} \tau_\theta \\ \dot{x} &= \frac{d\, f_x}{d\,\alpha}\dot{\alpha} \quad \dot{y} = \frac{d\, f_y}{d\,\alpha}\dot{\alpha} \quad \dot{\theta} = \frac{d\, f_\theta}{d\,\alpha}\dot{\alpha} \end{aligned} \tag{16}$$

C. Friction modeling and simulation

Static and Coulomb friction play an important role in the system. They are due to both the rings within the lift cylinder (that avoids any oil leakage even at high pressures) and the bearing shells between the levers. To include these dissipative phenomena in the model, an approach similar to the one presented in [8] and in [9] is followed.

First note that the Coulomb friction due to the cylinder rings and the one due to the bearing shells act in parallel, therefore their amplitude can be added and a unique Coulomb friction torque can be considered. The Coulomb friction torque τ_{bc} depends on the sign of the speed according to the following relation (see [6]):

$$\tau_{bc} = K_{bc}\, \mathrm{sign}(\dot{\alpha})$$

where $K_{bc} \geq 0$ denotes the Coulomb friction amplitude. The viscous friction is described by:

$$\tau_{bv} = K_{bv}(\dot{\alpha}) \qquad K_{bv}(0) = 0 \qquad K_{bv}(\dot{\alpha})\, \dot{\alpha} \geq 0$$

where $K_{bv}(\dot{\alpha})$ is a nonlinear continuous function. Since the levers speed is relatively small, the viscous friction can be usually neglected, indeed its amplitude is very small if compared to the Coulomb friction one.

Since the function $M(\alpha)$ is strictly positive, according to (14), the sign of $\dot{\alpha}$ is the same as the sign of p_α. The torque τ_α in (16) is then the difference between the torques τ_p and the friction torques:

$$\tau_\alpha = \tau_p - \tau_{bc} - \tau_{bv} = \tau_p - K_{bc}\, \mathrm{sign}(p_\alpha) - K_{bv}(\dot{\alpha})$$

Let τ_s be:

$$\tau_s = -\frac{\partial H}{\partial \alpha}(\alpha, p_\alpha) + \tau_p - K_{bv}(\dot{\alpha}) + \frac{d\, f_x}{d\,\alpha} F_x + \frac{d\, f_y}{d\,\alpha} F_y + \frac{d\, f_\theta}{d\,\alpha} \tau_\theta$$

the time derivative of the momenta p_α in (16) becomes:

$$\dot{p}_\alpha = \tau_s - K_{bc}\, \mathrm{sign}(p_\alpha) \tag{17}$$

The equation (17) is not suitable to be simulated. Indeed when the variable p_α is zero and when $|\tau_s| \le K_{bc}$, the function $K_{bc}\,\mathrm{sign}(p_\alpha)$ causes a *sliding mode* condition (see [10]), the term $K_{bc}\,\mathrm{sign}(p_\alpha)$ starts switching at infinite frequency between the two values $\pm K_{bc}$ and $\dot{p}_\alpha$ is kept to zero. This condition cannot be precisely simulated by computers. The simulation algorithm presented in [8] faces this problem and allows to achieve fast and precise simulations. The key idea of this algorithm is to substitute the term $K_{bc}\,\mathrm{sign}(p_\alpha)$ with its *equivalent control* (see [10]) when $|\tau_s| \le \gamma\, K_{bc}$ and $p_\alpha = 0$. The coefficient $\gamma \ge 1$ is the ratio between the static and the Coulomb friction amplitudes. By this way, computing the time derivative of the momenta p_α as:

$$\dot{p}_\alpha = \begin{cases} \tau_s - K_{bc}\,\mathrm{sign}(p_\alpha) & \text{if} \quad p_\alpha \neq 0 \\ 0 & \text{if} \quad p_\alpha = 0 \text{ and } |\tau_s| \le \gamma\, K_{bc} \\ \tau_s - \gamma\, K_{bc}\,\mathrm{sign}(\tau_s) & \text{if} \quad p_\alpha = 0 \text{ and } |\tau_s| > \gamma\, K_{bc} \end{cases}$$

it is possible to simulate both the static and the Coulomb friction avoiding the sliding mode condition. Moreover the amplitudes of the static and coulomb friction can also be time variable or speed dependent (as for the Stribeck Effect, see [6]).

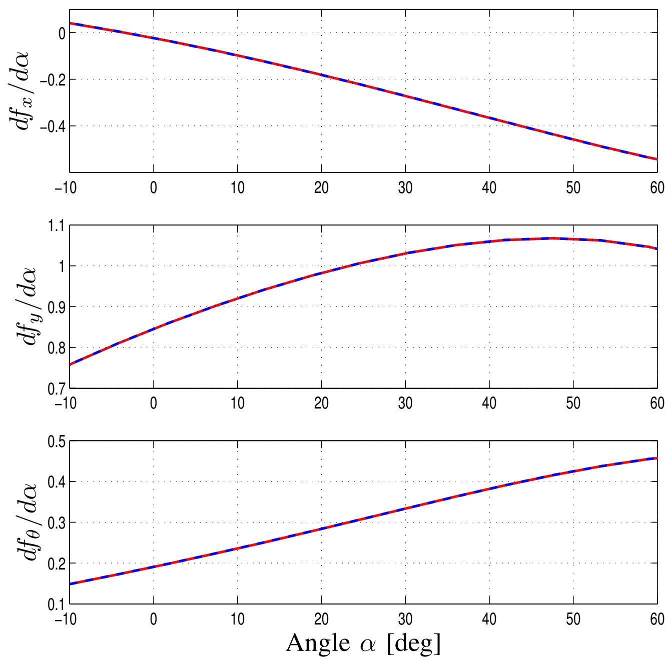

Fig. 3. Derivative of the functions (11) (dash-dotted) and their approximation by 4^{th} order polynomials (solid).

D. Model simplification for simulation

The dynamic model (16) is mathematically and energetically exact. However the functions (11) are rather complex since inverse trigonometric functions are involved, consequently their partial derivatives are huge and their computation slows down the simulations. To reduce the computational effort, it is possible to obtain a simplified model that maintains the hamiltonian framework. This guarantees that the approximated model keeps its physical meaning and it does not hide any false energy generation/dissipation process.

The basic idea is to interpolate the derivative of the functions (11) by polynomials. Then, from (13), the function $M(\alpha)$ becomes a polynomial and all the functions that appears in (16) are polynomials. The order of the polynomials can be chosen to obtain the best fitting with the analytical function. Figure 3 shows the derivative of the functions (11) and their approximation by 4^{th} order polynomials: the approximations almost overlap the analytical functions.

IV. EXPERIMENTAL SETUP AND SIMULATION RESULTS

The experimental setup consists in a three point hitch connected to a farm tractor. To avoid oscillations of the tractor due to the tires and the hitch movement, the tractor chassis was fixed to the ground. A standard implement was attached to the hitch. Experiments with different load weights and positions were measured. The following signals were measured: the two control currents i_R and i_F, the lift cylinder pressure P and the arm angular position α. The experiments were made by giving step input currents of different amplitudes to rise and fall the implement at different speeds.

Figures 4-11, compare the experimental measurements with the corresponding simulations of the proposed model. The comparisons are referred to both upward and downward movements, the results shown in Figs. 4-7 refer to a experiment with heavy load, the results in Figs. 8-11 correspond to a “light” load. For a non-disclosure agreement the axis are normalized, however to allow a correct comparison the constants T_0 and P_{max} are the same for all the plots.

Excluding the mass and the position of the implement, the coulomb friction amplitude K_{bc} is the only system's parameter that does change when the load is varied, the hydraulic parameters are the same in all the simulations. The amplitude K_{bc} is smaller when the load is lighter, this is due to the reduction of the friction forces on the bearing shells when the load is lighter.

All the comparisons show a good matching between the experimental and the simulated data. The lift arm position α is well replicated in all the comparisons, the pressure waves in the cylinder are quite similar to the experimental data.

V. CONCLUSIONS

A dynamic model of an electro-hydraulic three point hitch for farm tractors has been proposed. The model allows fast and precise simulations and therefore can be used for the development and the validation of control strategies by simulations and hardware in the loop experiments. The proposed model has been validated comparing the simulation results with experimental measurements. The modeling technique based on Hamiltonian systems and power-ports has demonstrated to be a powerful tool for modeling the dynamic systems.

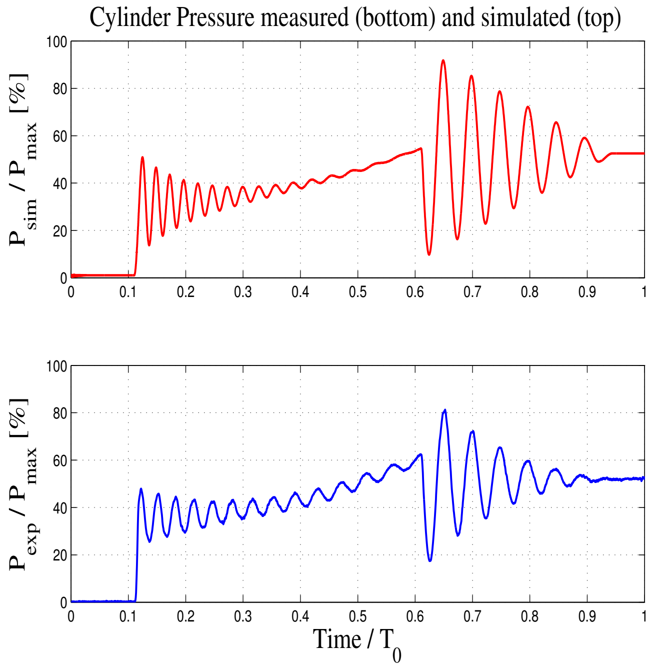

Fig. 4. Cylinder pressure P measured (bottom) and simulated (top) for an hitch lift with heavy load.

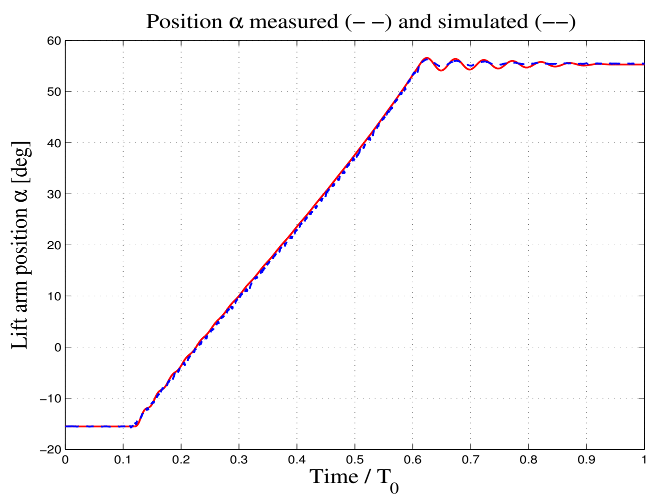

Fig. 5. Lift arm angular position α measured (dashed) and simulated (solid) for an hitch lift with heavy load.

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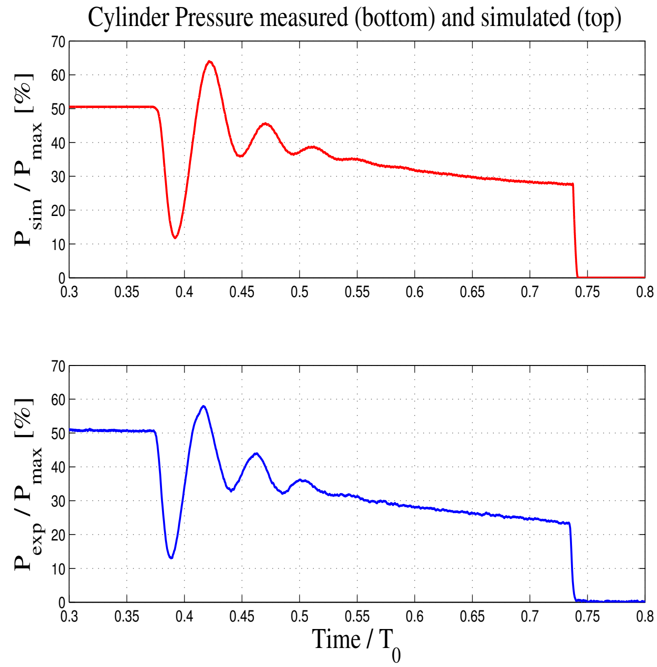

Fig. 6. Cylinder pressure P measured (bottom) and simulated (top) for an hitch fall with heavy load.

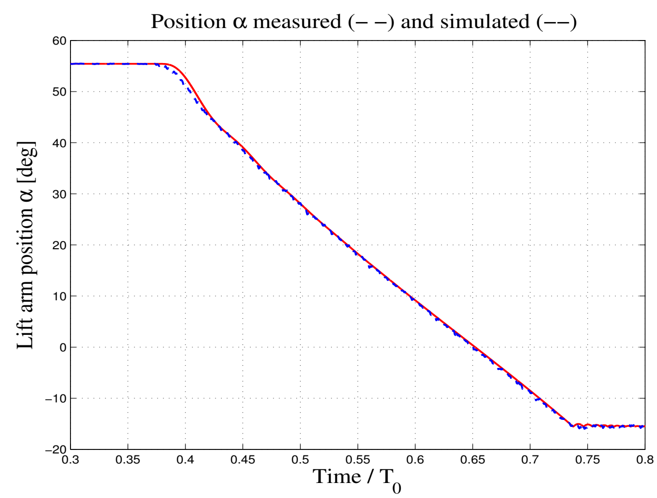

Fig. 7. Lift arm angular position α measured (dashed) and simulated (solid) for an hitch fall with heavy load.

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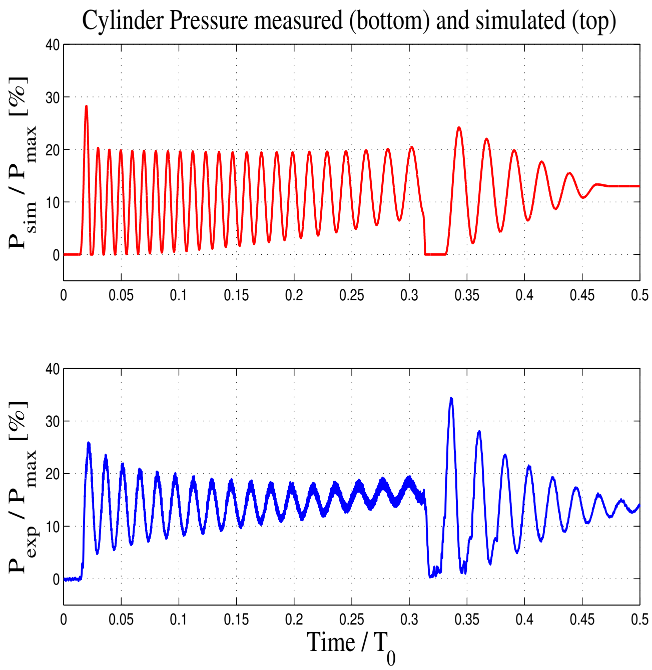

Fig. 8. Cylinder pressure P measured (bottom) and simulated (top) for an hitch lift with light load.

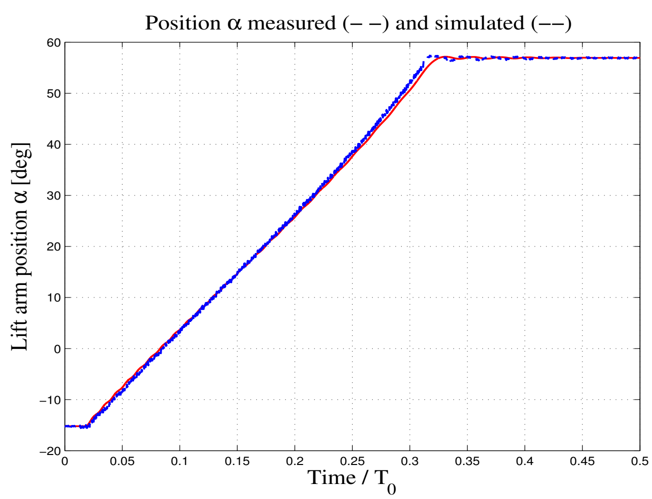

Fig. 9. Lift arm angular position α measured (dashed) and simulated (solid) for an hitch lift with light load.

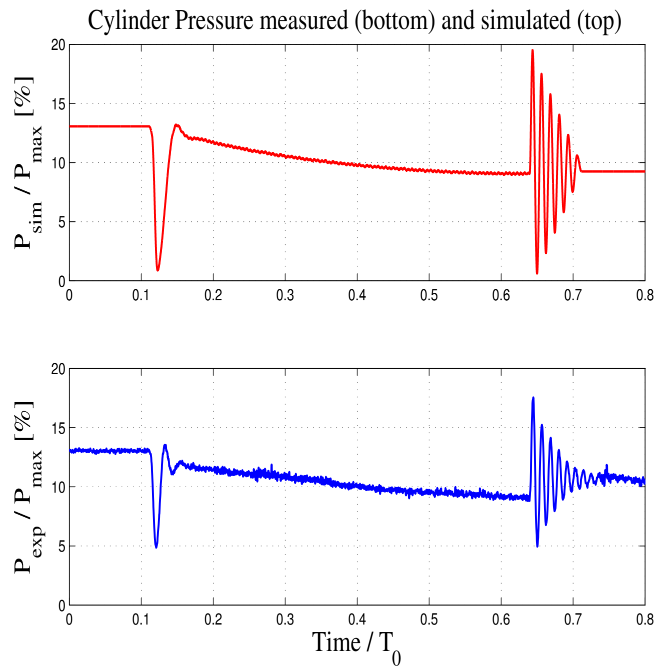

Fig. 10. Cylinder pressure P measured (bottom) and simulated (top) for an hitch fall with light load.

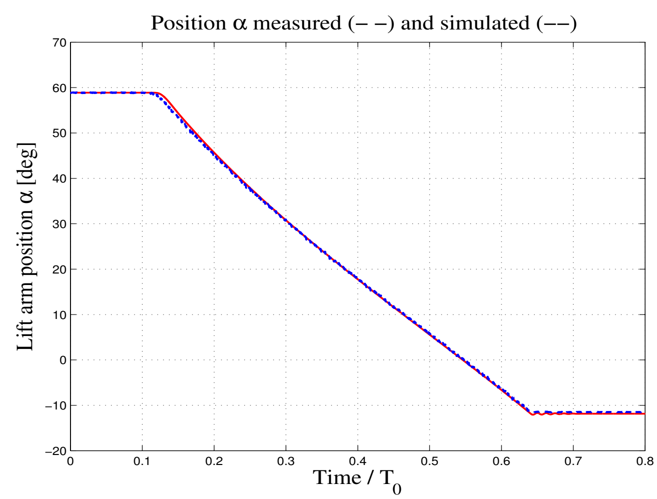

Fig. 11. Lift arm angular position α measured (dashed) and simulated (solid) for an hitch fall with light load.