Optimal Rotor Flux Shape for Multi-phase Permanent Magnet Synchronous Motors

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Abstract—In the paper the Power-Oriented Graphs (POG) technique is used for modeling $m$-phase permanent magnet synchronous motors and a study on the optimal rotor flux is given. The POG model shows the “power” internal structure of the considered electrical motor: the electric part interacts with the mechanical part by means of a “connection” block which neither stores nor dissipates energy. The dynamic model of the motor is as general as possible and it considers an arbitrary odd number of phases. The rotor flux is analyzed, in particular in order to minimize the currents needed for the torque generation, and phases. The rotor flux is analyzed, in particular in order to find the optimal shape of the rotor flux minimizing losses (gear reductors, etc.), i.e. all the 2-port elements dissipate energy (springs, masses, dampers, etc.), i.e. all the BG 2-port elements. The obtained POG model is very compact, as simple and puts in evidence the “power” internal structure in Matlab/Simulink and the presented simulation results validate the machine model and the rotor flux choice.

I. INTRODUCTION

The dynamic model of the multi-phase permanent magnet synchronous motors is known in literature obtained using classical mathematical methods. In this paper the dynamic model of these motors has been obtained using a Lagrangian approach in the frame of the Power-Oriented Graphs (POG) technique and, for the sake of generality, a generic periodic shape for the rotor flux has been considered. The obtained POG model is very compact, simple and puts in evidence the “power” internal structure of the motor. Using the POG approach and a Concordia-like transformation, the torque vector of the motor assumes a very simple structure which has been analyzed to find the optimal shape of the rotor flux minimizing the electrical power dissipation. The paper is organized as follows. Sec. II gives the basic properties of the POG modeling technique. Sec. III shows the details of POG dynamic model of the $m$-phase synchronous motors and the optimal shape of the rotor flux. Finally, in Sec. IV some simulation results are reported.

A. Notations

Row matrices will be denoted as follows:

$$[[R_{1:n}]] = \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix},$$

column and diagonal matrices as:

$$[[R_{i,j}]] = \begin{bmatrix} R_{11} & R_{12} & \ldots & R_{1m} \\ R_{21} & R_{22} & \ldots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \ldots & R_{nm} \end{bmatrix},$$

and full matrices as:

$$i \cdot [R_{i,j}] = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix},$$

$$[R_{1:n}] = \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}.$$

The symbol $\sum_{n=a:d}^{b} c_n = c_a + c_{a+d} + c_{a+2d} + \ldots$ will be used to represent the sum of a succession of numbers $c_n$ where the index $n$ ranges from $a$ to $b$ with increment $d$ that is, using the Matlab symbology, $a = [a : d : b]$.

II. THE BASES OF POWER-ORIENTED GRAPHS

The POG technique [1] is a graphical modeling technique similar to Bond Graph (BG) [2], [3] and Energetic Macroscopic Representation (EMR) [4]. These techniques use the “power interaction” between subsystems as basic element for modeling. The two basic blocks used in the POG technique are shown in Fig. 1: the “elaboration block” (e.b.) and the “connection block” (c.b.). There is no restriction on the vector variables $x$ and $y$ other than the fact that their inner product $\langle x, y \rangle = x^T y$ must have the physical meaning of a “power”. The e.b. is used for modeling all the physical elements that store and/or dissipate energy (springs, masses, dampers, etc.), i.e. all the 1-port elements (capacitors $C$, inertias $I$ and resistor $R$) used in the BG technique. The c.b. is used for modeling all the physical elements that “transform the power” without losses (gear reductors, etc.), i.e. all the BG 2-port elements (transformers $TR$, gyrators $GY$, modulated transformers MTR and modulated gyrators MGY). The summation element at the top of the e.b. is used for modeling all the 3-port connection elements (0-junction and 1-junction) of the BG technique. More details on Power-Oriented Graphs are reported in [1], [5] and [6].

Fig. 1. The POG basic blocks: the elaboration block (e.b.) on the left and the connection block (c.b.) on the right.