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Efficient simulation of Static and Dynamic friction for Automotive Applications

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Abstract: In this paper, a technique for modeling frictional phenomena among several bodies that move relatively is proposed with the purpose of optimizing the dynamic simulation of a complex system such as the driveline of a vehicle based on a dual clutch transmission. The proposed method considers together the inertial and frictional properties of the different subsystems, but decouples a main dynamics, that describes their average motion and does not depend on friction, from a proper number of relative dynamics. In this way, for each of these dynamics a standard static friction models can be used, leading to a very efficient numerical simulation. In this sense, the method adopted can be considered an enhancement of Karnopp model.

Keywords: Friction, Dynamic modeling, Numerical simulation, Automotive.

1. INTRODUCTION

In many applications, friction (and in particular Coulomb and static friction) is only a side effect of the mechanical implementation, that produces undesired behaviors and energy losses. For this reason, it is desirable to reduce as much as possible its influence on the mechanical system and, in many cases, Coulomb and static friction can be neglected. Conversely, in some applicative fields, such as the automotive field, Coulomb and static friction play a fundamental role. Consider for instance the case of clutches and similar devices, that exploit the capability of the friction of nullify the relative velocities between two contacting surfaces to synchronize different shafts and modulate the power transmission from the engine to the wheels. The dry clutch is adopted in almost all commercial cars, and also the synchronizers of traditional gearbox are based on Coulomb friction. For a complete overview about the use of clutches and therefore friction within automotive transmission refer to Jurgen (2000).
The digital simulation of such systems may be computationally complex and time-consuming because of the hard nonlinearities of the friction characteristic, that produces chattering, when the speed is zero or close to zero. In particular, in the case of clutches, their behavior changes dramatically according to the fact that they are engaged (relative speed null and static friction between the two plates) or not (in this case Coulomb friction must be considered). For this reason, many authors prefer to switch among different models (with different sets of equations) according to the state of the clutch (and therefore the type of friction, static or dynamic, acting on the system), see M.Kulkarni et al. (2007); Zanasi et al. (2008); Jiang et al. (2009). In this paper, a technique that allows to simulate (without the need of switching among several models) dynamic and static friction in complex systems, where frictional phenomena arise among several bodies that move relatively, is firstly presented; then the proposed method is applied to a double-clutch that synchronizes the input shaft connected to the engine with the two shafts linked to the gearbox.

2. FRICTION MODELS

A number of models for friction simulation and compensation have been proposed in the literature, ranging from classical static models (see Karnopp (1985); Armstrong-Hlouvry et al. (1994); Olsson et al. (1998)) to more complex dynamic models, such as the Dahl model by Dahl (1968), the Bliman and Sorine model by Bliman and Sorine (1995), and the Lugre model by de Wit et al. (1995), among many others. In particular, it is necessary to highlight that in all the applications where friction at rest plays an important role it is necessary to consider, besides the dynamic friction due to the motion, i.e. Coulomb and viscous friction, the frictional phenomena that arise when the velocity is null. The *stiction* (static friction) counteracts the external forces below a certain threshold and therefore prevents the object from moving. For this reason, it is clear that the friction at rest cannot be described as a function of only velocity but depends also on the applied forces. Moreover it is also clear that frictional phenomena and inertial phenomena are closely related, and therefore they should be modeled together.
A simple, although quite complete description of the friction force acting on a system with velocity v is

$$F_f = \begin{cases} F(v) & \text{if } v \neq 0 \\ F_e & \text{if } v = 0 \text{ and } |F_e| < F_s \\ F_s \text{sgn}(F_e) & \text{otherwise} \end{cases} \quad (1)$$

where $F(v)$ is an arbitrary function that reproduces the classical characteristic of dynamic friction, F_e are the external forces acting on the system, and F_s is the maximum value of the stiction. A quite commonly used expression of the dynamic friction force is

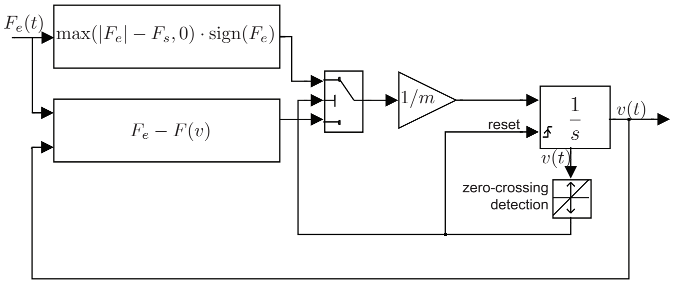


Fig. 1. Sketch of the simulink block-scheme for the efficient simulation of the dynamics of a mass m affected by static and dynamic friction.

$$F(v) = \left(F_c + (F_s - F_c)e^{-|v/v_s|^\alpha}\right)\mathrm{sgn}(v) + F_v v \qquad (2)$$

where the parameters F_c and F_v denote respectively the minimum level of Coulomb friction and the the viscous friction coefficient, while v_s and α are empirical parameters to be properly set according to the experimental characteristic of the friction, see Armstrong-Hlouvry et al. (1994). Note that some of the coefficients F_s, F_c, F_v may be zero according to the specific phenomena to be modeled. For instance in the model of a dry clutch, the viscous friction between the two plates is negligible. Moreover, the coefficient may be not a constant but a function of the normal force F_n applied to the surfaces in contact. In general, the instantaneous values of force friction are

$$F_c(t) = \mu_c F_n(t), \qquad F_s(t) = \mu_s F_n(t).$$

where μ is the coefficient of friction. One of the major simulative problem that arises when the model (1) is adopted is the need of detecting when the velocity becomes zero. A simple remedy suggested by Karnopp (1985) consists in defining an interval in which the velocity v is forced to be zero and the friction force is computed as a saturated version of the external force. A commonly recognized drawback of the Karnopp model is the fact that it is strongly coupled with the rest of the dynamic system, since the external force is an input of the model. But, as already mentioned, this is not a weak point of the model, but a simple consequence of the fact that frictional phenomena, in particular at rest, are closely connected with the inertial behavior of the systems that they affect. As a consequence they should modeled and simulated together. In this perspective, the simulation of a mass m subject to an external force F_e and to the friction, as defined in (1), can be performed by numerically integrating the dynamic equation

$$m\dot{v} = F_e - F_f \qquad (3)$$

considering two different expressions of the net force applied to the system according to the value of v, i.e.

$$m\dot{v} = \begin{cases} F_e - F(v), & \text{if } v \neq 0 \\ \max(|F_e| - F_s, 0) \cdot \mathrm{sign}(F_e), & \text{if } v = 0 \end{cases} \qquad (4)$$

The switch from the first expression to the second one must occur when a zero-crossing of the velocity v is detected. At the same time v must be hold to zero until the external force overcomes the stiction value F_s. In this way, the chattering and other simulative problems that may arise at zero velocity because of the hard non-linearity of the friction are prevented, since as soon as a zero-crossing is detected the velocity is maintained at zero. In Fig. 1 a sketch of the simple simulative scheme corresponding to the above procedure is reported.

The main advantages of this scheme with respect to the other models of friction, and in particular dynamic models, is that it does not require additional parameters (such as stiffness and damping coefficients of bristles in the Lugre model, see de Wit et al. (1995)), whose value is hardly estimable from experimental data. Nevertheless, although quite simple and computationally efficient, the scheme of Fig. 1 has a significant drawback. As a matter of fact it is not directly applicable to systems composed by several moving bodies that interact by friction, as in case of a clutch, where the friction is exerted between two rotative elements. Aim of this paper is to extend the use of the model of Fig. 1 to this class of systems.

3. SIMULATION OF MULTI-BODY SYSTEMS INTERACTING BY FRICTION

In this section, the model illustrated in previous section is adapted to correctly simulate the behavior of systems composed by a number of objects that interact in pairs by means of frictional interfaces. Since, the goal of this research concerns the simulation of clutches used in the automotive field, rotative systems are taken into account, but the same considerations hold true for translating systems.

3.1 Two masses model

Given the system of Fig. 2(a), composed by two rotating bodies with inertia J_1 and J_2 respectively, described by the dynamic model

$$\begin{cases} J_1\dot{\omega}_1 = F_1 - \tau_{12}(\omega_1 - \omega_2) \\ J_2\dot{\omega}_2 = F_2 + \tau_{12}(\omega_1 - \omega_2) \end{cases} \qquad (5)$$

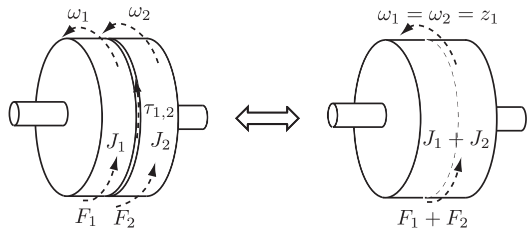
where ω_i and F_i are the angular velocity and the external torque related to the i-th mass and $\tau_{i,i+1}(\cdot)$ denotes the friction at the interface between the i-th and the $(i+1)$-th object, it is possible to obtain the same formulation as in (3), by means of a proper congruent state space transformation. Firstly, it is convenient to rewrite the system (5) in a matrix form as

$$\boldsymbol{J}\dot{\boldsymbol{\Omega}} = \boldsymbol{F} - \boldsymbol{D}^T\boldsymbol{\tau}(\boldsymbol{D\Omega}) \qquad (6)$$

with

$$\boldsymbol{J} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}, \quad \boldsymbol{\Omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \quad \boldsymbol{D} = [1 \quad -1]$$

and $\boldsymbol{\tau} = \tau_{12}$. Then, by applying the transformation matrix $\boldsymbol{T}$ proposed by Zanasi et al. (2001)



(a) (b)

Fig. 2. Two rotating inertias interacting by friction (a) and equivalent system obtained when $z_2 = 0$ (b).

$$\boldsymbol{\Omega} = \boldsymbol{T}\boldsymbol{z}, \quad \boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \frac{J_1\omega_1 + J_2\omega_2}{J_1+J_2} \\ \omega_1 - \omega_2 \end{bmatrix}, \quad \boldsymbol{T} = \begin{bmatrix} 1 & \frac{J_2}{J_1+J_2} \\ 1 & \frac{-J_1}{J_1+J_2} \end{bmatrix}$$

the original system (6) can be transformed in

$$\underbrace{\boldsymbol{T}^T\boldsymbol{J}\boldsymbol{T}}_{\boldsymbol{J}_T}\dot{\boldsymbol{z}} = \underbrace{\boldsymbol{T}^T\boldsymbol{F}}_{\boldsymbol{F}_T} - \underbrace{(\boldsymbol{D}\boldsymbol{T})^T}_{\boldsymbol{D}_T}\boldsymbol{\tau}(\boldsymbol{D}\boldsymbol{T}\boldsymbol{z}) \tag{7}$$

with

$$\boldsymbol{J}_T = \begin{bmatrix} J_1+J_2 & 0 \\ 0 & \frac{J_1J_2}{J_1+J_2} \end{bmatrix}, \boldsymbol{F}_T = \begin{bmatrix} F_1+F_2 \\ \frac{J_2F_1 - J_1F_2}{J_1+J_2} \end{bmatrix}, \boldsymbol{D}_T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Therefore, the expression of the dynamic system in the new state variables z_i becomes

$$\begin{bmatrix} J_1+J_2 & 0 \\ 0 & \frac{J_1J_2}{J_1+J_2} \end{bmatrix}\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} F_1+F_2 \\ \frac{J_2F_1 - J_1F_2}{J_1+J_2} \end{bmatrix} - \begin{bmatrix} 0 \\ \tau_{12} \end{bmatrix}$$

where it is possible to recognize a main dynamics

$$(J_1+J_2)\dot{z}_1 = F_1 + F_2 \tag{8}$$

describing the average motion of the system, in which the internal torque due to the friction between the two inertias does not appear (and consequently simulation problems are not present), decoupled from the relative dynamics that describes the differential motion of the two bodies:

$$\frac{J_1J_2}{J_1+J_2}\dot{z}_2 = \frac{J_2F_1 - J_1F_2}{J_1+J_2} - \tau_{12}(z_2). \tag{9}$$

Note that (9) has exactly the same structure of (3), with an inertia $J_R = \frac{J_1J_2}{J_1+J_2}$ subject to the equivalent torque $F_R = \frac{J_2F_1-J_1F_2}{J_1+J_2}$ and to the friction τ_{12}, that depends only on the (relative) velocity z_2. Therefore, in order to simulate the relative dynamics it is possible to adopt the scheme illustrated in the previous section.

Another interesting properties of the transformed system is that at rest the value of the friction is equal to F_R, with the obvious saturation to F_s. This result is well-known (see Serrarens et al. (2004) among many others), but it is worth noticing that the approach based on the congruence transformation $\boldsymbol{T}$ provides a systematic procedure for the computation of the friction at zero velocity. As a matter of fact, it is sufficient to solve the algebraic equation obtained from (9) by assuming $\dot{z}_2 = 0$. This result is quite intuitive since the static friction opposes the motion and, therefore, counteracts all the external torques in order to guarantee that the relative velocity remains zero. When $z_2 = 0$, the system is completely described by state variable z_1 and by the equation (8); as shown in Fig. 2(b) it behaves like a unique inertia $J_1 + J_2$, subject to the resultant of all the external torques. Note that, by substituting $\tau_{1,2} = \frac{J_2F_1-J_1F_2}{J_1+J_2}$ in both the equations of the original system (5) we just obtain (8).

3.2 Three masses model

Given the system composed by three rotating masses shown in Fig. 3, that are subject to external torques and friction between the contacting masses, as described by the system of differential equations

$$\begin{cases} J_1\dot{\omega}_1 = F_1 - \tau_{12}(\omega_1 - \omega_2) \\ J_2\dot{\omega}_2 = F_2 + \tau_{12}(\omega_1 - \omega_2) - \tau_{23}(\omega_2 - \omega_3) \\ J_3\dot{\omega}_3 = F_3 + \tau_{23}(\omega_2 - \omega_3) \end{cases} \tag{10}$$

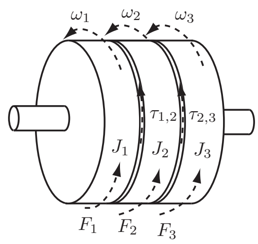


Fig. 3. Three rotating inertias interacting by friction (double dry clutch).

one may decouple the main dynamics that does not depends on the internal frictional torques from the other relative dynamics. By considering the state vector

$$\boldsymbol{z} = \begin{bmatrix} \frac{J_1\omega_1 + J_2\omega_2 + J_3\omega_3}{J_{Tot}} \\ \omega_1 - \omega_2 \\ \omega_2 - \omega_3 \end{bmatrix}$$

related to the vector of the velocities by $\boldsymbol{\Omega} = \boldsymbol{T}\boldsymbol{z}$, with

$$\boldsymbol{T} = \begin{bmatrix} 1 & \frac{J_2+J_3}{J_{Tot}} & \frac{J_3}{J_{Tot}} \\ 1 & -\frac{J_1}{J_{Tot}} & \frac{J_3}{J_{Tot}} \\ 1 & -\frac{J_1}{J_{Tot}} & -\frac{J_1+J_2}{J_{Tot}} \end{bmatrix}$$

where $J_{Tot} = J_1 + J_2 + J_3$, the system is translated into the form

$$\boldsymbol{J}_T\dot{\boldsymbol{z}} = \boldsymbol{F}_T - \boldsymbol{D}_T\boldsymbol{\tau}(\boldsymbol{D}_T^T\boldsymbol{z}) \tag{11}$$

with

$$\boldsymbol{J}_T = \begin{bmatrix} J_{Tot} & 0 & 0 \\ 0 & \frac{J_1(J_2+J_3)}{J_{Tot}} & \frac{J_1J_3}{J_{Tot}} \\ 0 & \frac{J_1J_3}{J_{Tot}} & \frac{(J_1+J_2)J_3}{J_{Tot}} \end{bmatrix}$$

$$\boldsymbol{F}_T = \begin{bmatrix} F_{T1} \\ F_{T2} \\ F_{T3} \end{bmatrix} = \begin{bmatrix} F_1+F_2+F_3 \\ \frac{-J_1(F_2+F_3)+(J_2+J_3)F_1}{J_{Tot}} \\ \frac{J_3(F_1+F_2)-(J_1+J_2)F_3}{J_{Tot}} \end{bmatrix}, \quad \boldsymbol{D}_T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By pre-multiplying both terms of (11) by $\boldsymbol{J}_T^{-1}$ one obtain the system of dynamic equation

$$\dot{z}_1 = \frac{F_{T1}}{J_{Tot}} \tag{12}$$

$$\dot{z}_2 = \left(\frac{1}{J_1} + \frac{1}{J_2}\right)(F_{T2} - \tau_{1,2}) - \frac{1}{J_2}(F_{T3} - \tau_{2,3}) \tag{13}$$

$$\dot{z}_3 = -\frac{1}{J_2}(F_{T2} - \tau_{1,2}) + \left(\frac{1}{J_2} + \frac{1}{J_3}\right)(F_{T3} - \tau_{2,3}) \tag{14}$$

where the friction terms $\tau_{1,2}$ and $\tau_{2,3}$ depend on z_2 and z_3 respectively. For their computation four distinct cases may occur:

(1) if $z_2 \neq 0$ and $z_3 \neq 0$ the friction depends only on the relative velocity, therefore $\tau_{1,2} = F(z_2)$, $\tau_{2,3} = F(z_3)$, where the function $F(\cdot)$ is defined by (2). Obviously, the characteristic parameters of the function $F(\cdot)$ may be different in the two cases;

(2) if $z_2 = 0$ and $z_3 \neq 0$, then $\tau_{2,3} = F(z_3)$ while the value of $\tau_{1,2}$ is determined by setting the right side of (13) equal to zero:

$$\tilde{\tau}_{1,2} = F_{T2} - \frac{J_1}{J_1+J_2}(F_{T3} - \tau_{2,3}(z_3)).$$

In this case, if $|\tilde{\tau}_{1,2}| < F_s$ then $\tau_{1,2} = \tilde{\tau}_{1,2}$ and $\dot{z}_2 = 0$; otherwise, $\tau_{1,2} = F_s \cdot \text{sign}(\tilde{\tau}_{1,2})$;

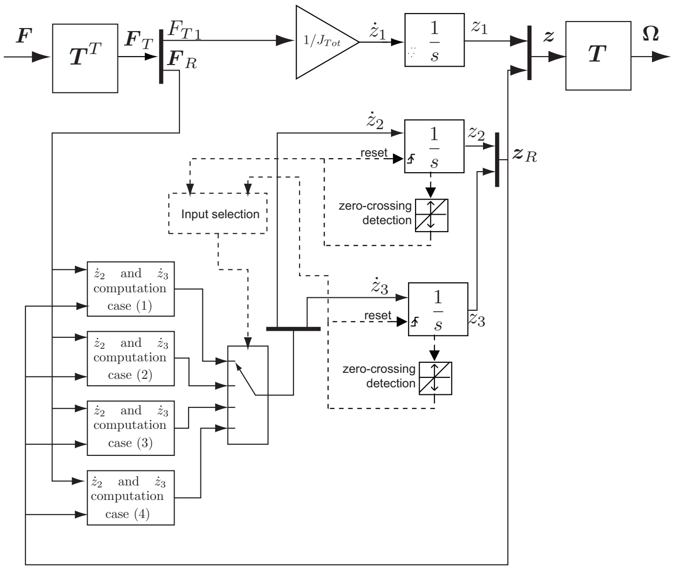


Fig. 4. Sketch of the simulative scheme of the system composed by three inertias.

(3) the case $z_2 \neq 0$ and $z_3 = 0$ is dual with respect to the previous one. Therefore, $\tau_{1,2} = F(z_2)$ while the value of $\tau_{2,3}$, determined by setting the right side of (14) equal to zero, is

$$\tilde\tau_{2,3} = F_{T3} - \frac{J_3}{J_2 + J_3}(F_{T2} - \tau_{1,2}(z_2)).$$

If $|\tilde\tau_{2,3}| < F_s$ then $\tau_{2,3} = \tilde\tau_{2,3}$ and $\dot z_3 = 0$; otherwise, $\tau_{2,3} = F_s \cdot \mathrm{sign}(\tilde\tau_{2,3})$;

(4) when $z_2 = 0$ and $z_3 = 0$, the friction values can be computed from (13) and (14). By setting the right side of both equations equal to zero we obtain a system whose solution is

$$\tilde\tau_{1,2} = F_{T2}, \qquad \tilde\tau_{2,3} = F_{T3}. \tag{15}$$

If $|\tilde\tau_{i,i+1}| < F_s$, $i = 1, 2$ then $\tau_{i,i+1} = \tilde\tau_{i,i+1}$ and $\dot z_{i+1} = 0$; otherwise, $\tau_{i,i+1} = F_s \cdot \mathrm{sign}(\tilde\tau_{i,i+1})$. Suppose that $|\tilde\tau_{1,2}| > F_s$; from a formal point of view, the solution found is not correct (since $\tau_{1,2} = \tilde\tau_{1,2}$ is not feasible), and should be recomputed by considering the term $\tau_{1,2} = F_s \cdot \mathrm{sign}(\tilde\tau_{1,2})$ as an input of the problem and solving the equation obtained from (14). Nevertheless, because of the continuity of the torques applied to physical systems and the typical small size of the integration step used in simulation, the error on $\tau_{2,3}$ is bounded and in general negligible. Moreover, the absolute value of $\tau_{2,3}$ is not important since it must simply guarantee that $\dot z_3 = 0$. Note that the possibility that both friction terms overcome the stiction threshold at the same time instant is rather unlikely. A similar argument holds true if $\tau_{2,3}$ is the first term that overcomes the stiction level. In conclusion, the solution expressed by (15) is acceptable for simulation purposes, provided that the derivative $\dot z_i$ is forced to zero when $|\tilde\tau_{i,i+1}| < F_s$.

The values of the friction substituted in (13) and (14) allow to correct simulate the dynamics of the system composed by three inertias, as shown in Fig. 4, where a schematic representation of the simulative model is reported. Note in particular that the system switches among the different expressions of $(\dot z_2, \dot z_3)$ that depend on the friction terms

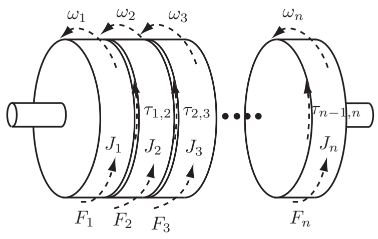


Fig. 5. A stack of n rotating inertias interacting by friction.

$\tau_{1,2}$, $\tau_{2,3}$. Nevertheless, the dynamic model of the system, and in particular its dynamical dimension, does not change when two contiguous masses are stuck (that is $z_i = 0$, if the $(i-1)$-th and i-th masses are considered) but, simply, the related variable z_i is forced to remain zero (by imposing $\dot z_i = 0$) until the external torques overcome the stiction level.

3.3 A stack of n frictional bodies

The approach shown in previous subsection for the simulation of a system composed by three masses can be generalized to an arbitrary number of bodies (for instance to model and simulate the behavior of a train of gears). Given a set of n bodies J_i, that interact in pairs, as illustrated in Fig. 5, the congruence transformation matrix

$$\boldsymbol{T} = \begin{bmatrix} 1 & \frac{J_2+\ldots+J_n}{J_{Tot}} & \frac{J_3+\ldots+J_n}{J_{Tot}} & \cdots & \frac{J_{n-1}+J_n}{J_{Tot}} & \frac{J_n}{J_{Tot}} \\ 1 & -\frac{J_1}{J_{Tot}} & \frac{J_3+\ldots+J_n}{J_{Tot}} & \cdots & \frac{J_{n-1}+J_n}{J_{Tot}} & \frac{J_n}{J_{Tot}} \\ 1 & -\frac{J_1}{J_{Tot}} & -\frac{J_1+J_2}{J_{Tot}} & \cdots & \frac{J_{n-1}+J_n}{J_{Tot}} & \frac{J_n}{J_{Tot}} \\ \vdots & \vdots & & & & \vdots \\ 1 & -\frac{J_1}{J_{Tot}} & -\frac{J_1+J_2}{J_{Tot}} & \cdots & -\frac{J_1+\ldots+J_{n-2}}{J_{Tot}} & \frac{J_n}{J_{Tot}} \\ 1 & -\frac{J_1}{J_{Tot}} & -\frac{J_1+J_2}{J_{Tot}} & \cdots & -\frac{J_1+\ldots+J_{n-2}}{J_{Tot}} & -\frac{J_1+\ldots+J_{n-1}}{J_{Tot}} \end{bmatrix}$$

with $J_{Tot} = \sum_{i=1}^{n} J_i$, leads to a dynamic system into the form (11), where the average dynamics

$$\dot z_1 = \frac{\sum_{i=1}^{n} F_i}{J_{Tot}} \tag{16}$$

is decoupled from relative dynamics

$$\dot{\boldsymbol{z}}_R = \boldsymbol{J}_R^{-1}\left(\boldsymbol{F}_R - \boldsymbol{\tau}(\boldsymbol{z}_R)\right), \tag{17}$$

being $\boldsymbol{z}_R = \boldsymbol{D}_T^T \boldsymbol{z} = [z_2, z_3, \ldots, z_n]^T$ the vector of relative velocities, $\boldsymbol{F}_R = \boldsymbol{D}_T^T \boldsymbol{F}_T = [F_{T2}, F_{T3}, \ldots, F_{Tn}]^T$, and $\boldsymbol{J}_R$ the submatrix obtained from $\boldsymbol{J}_T$ by eliminating the first row and the first column, i.e.

$$\boldsymbol{J}_T = \begin{bmatrix} J_{Tot} & \boldsymbol{0}_{n-1}^T \\ \boldsymbol{0}_{n-1} & \boldsymbol{J}_R \end{bmatrix}.$$

Note that the friction terms only depends on $\boldsymbol{z}_R$, and, as in the cases $n = 2$ and $n = 3$, $\tau_{i,i+1} = \tau(z_{Ri})$, $i = 2, \ldots, n$. In order to correctly (and efficiently) simulate the dynamics of the system, the friction torques acting between contacting inertias can be determined according to the following procedure. Let ξ denote the set of indexes i that correspond to zero relative velocities z_{Ri}, and $\bar\xi$ the remaining indexes for which $z_{Ri} \neq 0$, the friction torques between contiguous inertias that move relatively can be immediately obtained since they are function only of the velocity, therefore

$$\tau_{i,i+1} = F(z_{Ri}), \qquad i \in \bar\xi. \tag{18}$$

Conversely, when the relative velocities are null, the friction values $\tau_{i,i+1}$, $i \in \xi$, must be computed by considering

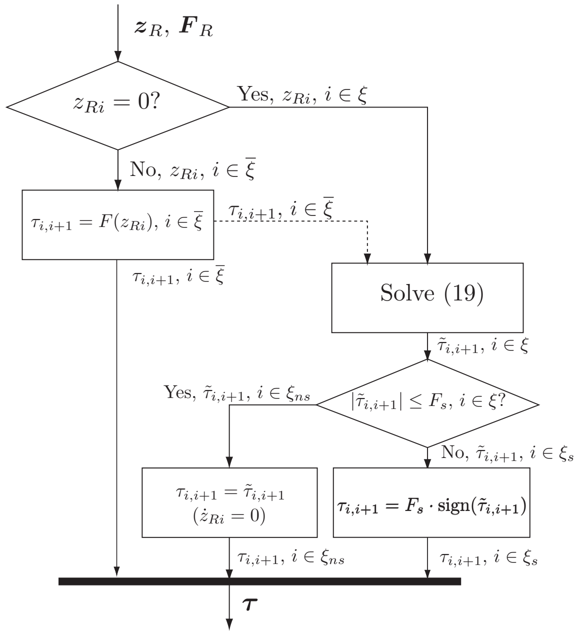


Fig. 6. Flowchart for the computation of friction in the case of n bodies.

the balance of all the torques acting on the system. In particular, as for the three masses system, such values can be calculated by setting the right terms of those equations of the system (17) for which $z_{Ri} = 0$ equal to zero. The expression of the friction terms $\tau_{i,i+1}$, $i \in \xi$ results

$$\tilde{\boldsymbol{\tau}}_{(\boldsymbol{\xi})} = \boldsymbol{F}_{R(\xi)} + \left(\left(\boldsymbol{J}_R^{-1}\right)_{(\xi,\xi)}\right)^{-1} \cdot \left(\boldsymbol{J}_R^{-1}\right)_{(\xi,\bar{\xi})} \cdot \left(\boldsymbol{F}_{R(\bar{\xi})} - \boldsymbol{\tau}_{(\bar{\xi})}\right) \tag{19}$$

where the subscript (p), with p set of indexes, applied to a vector $\boldsymbol{v} = [v_1, v_2, \ldots, v_n]^T$ denotes the new vector obtained by considering only the components v_i, with $i \in p$, while the subscript (p, q) applied to a matrix $\boldsymbol{A} = [a_{i,j}]$ stands for the sub-matrix that includes only the elements $a_{i,j}$, with $i \in p$ and $j \in q$. Note that the elements of the vector $\tilde{\boldsymbol{\tau}}_{(\boldsymbol{\xi})}$ represent the real friction values only if

$$|\tilde{\tau}_{i,i+1}| \le F_s, \qquad i \in \xi. \tag{20}$$

For the terms that meet condition (20)

$$\tau_{i,i+1} = \tilde{\tau}_{i,i+1} \text{ and } \dot{z}_{Ri} = 0. \tag{21}$$

On the contrary, if $|\tilde{\tau}_{i,i+1}| > F_s, i \in \xi$, the related friction values must be saturated to F_s, i.e.

$$\tau_{i,i+1} = F_s \cdot \mathrm{sign}(\tilde{\tau}_{i,i+1}). \tag{22}$$

In this case, as already discussed in Sec. 3.2, the solution is not exact, but it approximates the real value.

In the general case $n > 3$, the computation of $\tau_{i,i+1}$, $i = 1, \ldots n-1$ according to all the possible configurations of the system, and in particular to the fact that the relative velocities z_{Ri} are zero or not, may become prohibitive. As a matter of fact, the different cases are 2^{n-1}. As a consequence, for high values of n, it is convenient to directly implement the equations (18), (19), (21), (22) for the computation of the friction (see the flow diagram of Fig. 6) and consider the dynamic equations (16) and (17). In Fig. 7 the block-scheme representation of the simulative model for the system composed by n inertias is shown. Note, in particular, that by means of the power port defined by the pair $(\boldsymbol{F}, \boldsymbol{\Omega})$, it is possible to connect the system to the models of the other mechanical elements

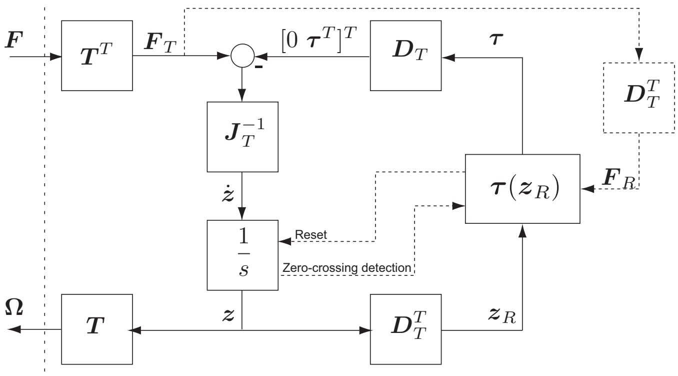


Fig. 7. Block-scheme representation of the system composed by n bodies, interacting by friction.

composing the plant under investigation, as illustrated in the example considered in the following section.

4. SIMULATION OF A DOUBLE CLUTCH

The working principle of the clutch used in dual clutch transmission (DCT) configurations is shown schematically in Fig. 8. It consists of two clutches that are arranged concentrically and whose friction plates are linked to the same shaft. Therefore, this system can be modeled as the three mass system of Sec. 3.2, where ω_2, F_2 are the velocity and the torque of input shaft connected to the engine, while ω_1, F_1 and ω_3, F_3 are the velocities and the torques of the shafts linked to the gearbox. The state of the clutches (engaged, slipping, or open) and therefore the power transmission from the engine to the gearbox can be separately changed by modulating the pressure between the two plates of each clutch. In order to simulate its behavior, the clutch has been inserted in a very simplified model of the vehicle, that takes into account only few elements of the drive-line. In Fig. 9 a power-oriented graph representation of the system is shown, see Morselli and Zanasi (2006). The vehicles is modeled as an equivalent inertia J_v subject to a friction torque $b_v\omega_v$, while in k_1, b_1, and k_3, b_3 are lumped all the elastic and dissipative effects of the drive-line. The gains R_1 and R_3 take into account the two different transmission ratios of odd and even gears. Finally, the engine is modeled as a constant torque input (400 Nm). In Fig. 10 the velocities of the shafts connected to the double clutch are reported. The two clutches are engaged alternatively by properly acting on the pressure of the two plates composing each of them and modulating in this way the related values F_c and F_s of Coulomb and static friction. When a clutch is activated

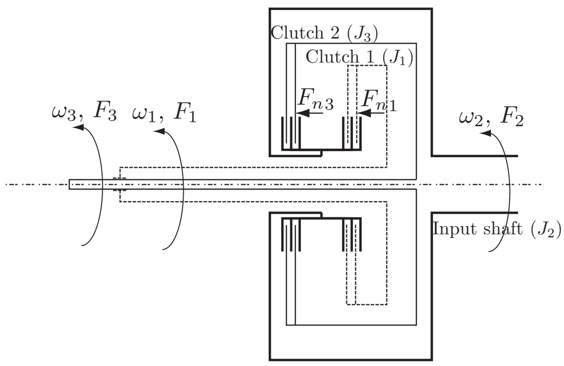


Fig. 8. Simplified model of a double clutch.

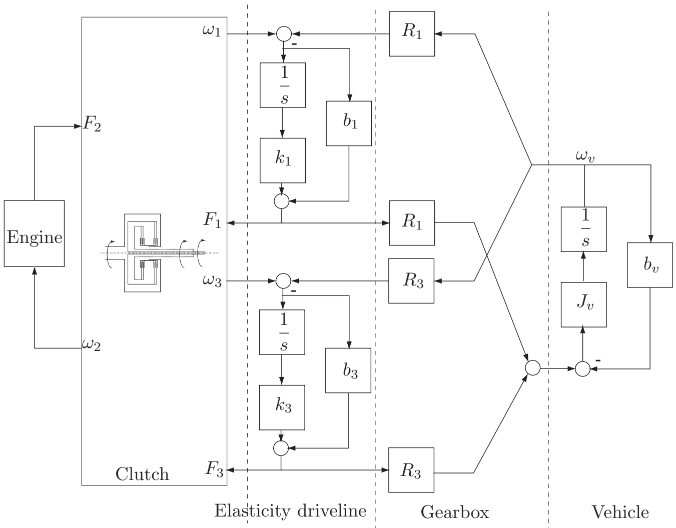
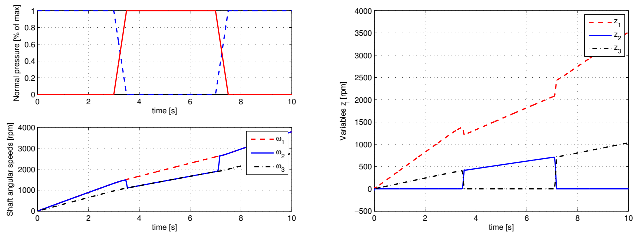


Fig. 9. POG representation of the double clutch inserted in a simplified model of a vehicle.



(a) (b)

Fig. 10. Angular velocities of the shafts connected to the double clutch (a) and related variables z_i (b).

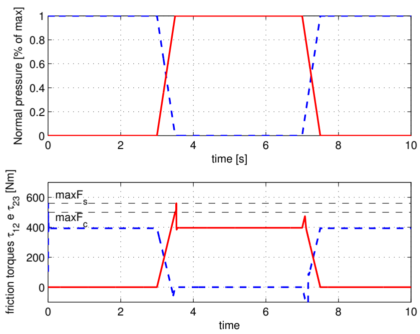


Fig. 11. Friction between the plates of the two clutches.

(and the other released) the friction works so that its velocity reach the velocity of the input shaft (and the related variable z_i goes to zero). In Fig. 11 the values of the friction are shown. Clearly, when a clutch is engaged and the other completely released, a friction term equals the engine torque (that is transmitted to the wheels), while during slipping phases the friction depends on the Coulomb coefficient. Therefore, in order to guarantee a continuous transmission of the power from the engine to the wheels during a shift, it is necessary to properly control the normal pressure that modulates F_c. From a simulative point of view, it is worth noticing that no chattering affects the friction. This makes the simulation very efficient: on a standard personal computer equipped with an Intel Core 2 Duo CPU at 2.26 GHz, and 4GB of ram, the Simulink simulation (of 10s) takes 0.98s.

5. CONCLUSIONS

In this paper, a systematic approach for modeling complex systems, composed by n bodies that move relatively and are affected by dynamic and static friction, is adopted. This technique considers together the inertial and frictional properties of the system, but decouples a main dynamics, that describes its average motion and does not depend on friction, from $n-1$ relative dynamics. This simplifies the initial problem and allows the use of standard static friction models, without the need of setting all the characteristic parameters of dynamic friction models that do not own a clear (macroscopic) physical interpretation. In particular, the proposed method is quite efficient from a simulative point of view, since it does not require the switching among several dynamic models and it does not produce chattering when the velocities are zero or close to zero. The effectiveness and efficiency of this friction model has been proved by considering the simulation of a dual clutch transmission system.

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