Systems and Control Theory
Master Degree in ELECTRONICS ENGINEERING
(http://www.dii.unimore.it/∼lbiagiotti/SystemsControlTheory.html)

Exercises #10

Matlab start

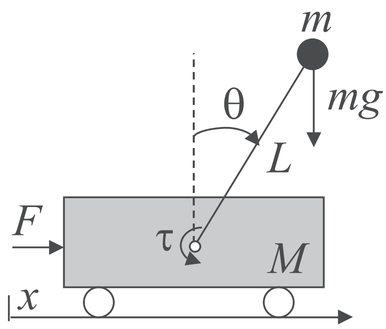
The exercises are carried out under Linux operating system. In order to start the MATLAB program and create the working directory `surname.name`, where all the MATLAB and SIMULINK files must be included, follow the procedure here reported:

1. Login with username and password used for the Unimore e-mail.
2. Open a `Terminal`.
3. Create the working directory and enter it with the commands
   `mkdir cognome.nome`
   `cd cognome.nome`
4. Open MATLAB with the command `matlab_R2006b`
5. Carry out the exercises, by using M-file, M-functions and Simulink schemes. Remember that the main file must be named `exercise.m` (in the first line of this file specify first name and surname, properly commented).

Text of the exercises

Design an M-file (exercise.m) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

Let's consider the cart-pole system shown in the figure. By assuming that the mass of the cart is M and the mass m of the inverted pendulum is lumped at the end of massless rigid rod, the model of the system is:



$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \dfrac{m\,L\,x_4^2\,\sin(x_3) - m\,g\,\sin(x_3)\,\cos(x_3) + u}{M + m\,(\sin(x_3))^2} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \dfrac{-m\,L\,x_4^2\,\sin(x_3)\cos(x_3) + (M+m)\,g\,\sin(x_3) - u\,\cos(x_3)}{L\,(M + m\,(\sin(x_3))^2)} \end{cases}$$

where the state variables are

$$\mathbf{x} = [\,x_1\ \ x_2\ \ x_3\ \ x_4\,]^T = [\,x\ \ \dot{x}\ \ \theta\ \ \dot{\theta}\,]^T$$

and the input $u = F$ is the force applied to the cart.

1. By considering the numerical values $m = 0.1$ kg, $M = 1$ kg, $L = 1$ m, $g = 9.81$ m/s^2, build the Simulink model that allows to simulate the dynamics of the nonlinear system. Assume that the measured variables are the cart position x and the pendulum angular position θ, i.e. $y = [x,\,\theta]^T$.

2. Linearize the system about the equilibrium point $x_e = [\bar{x}\ 0\ 0\ 0]^T$, where $\bar{x}$ is a generic cart position (in the Simulink model assume $\bar{x} = 1$), corresponding to the input $u_e = 0$ and analyze its stability.

3. Design a LQ regulator for the linearized system that minimizes the performance index

$$J = \frac{1}{2}\int_0^\infty 5\delta x^2(t) + \delta\theta^2(t) + \delta u^2(t)\,dt \tag{1}$$

   Simulate the evolution of the controlled (linear) system from the initial conditions $\delta x_0 = [0\ 0\ 0.55\ 0]^T$ (duration of the simulation 6s). Plot in the same figure (2 subplots) the evolution of $\delta x(t)$ (1st subplot) and $\delta\theta(t)$ (2nd subplot).

4. After having analyzed the observability of the system, design an asymptotic state estimator and insert it in the Simulink scheme. Plot in a new figure the evolution of $\delta x(t)$ and $\delta\theta(t)$ obtained with the same initial conditions of previous point and in a new figure compare the components of the actual state and those of the estimated state.

5. Apply the dynamic output feedback to the nonlinear system and simulate the evolution from the initial condition $x_e + \delta x_0$. Plot in a new figure the evolution of $x(t)$ and $\theta(t)$.

6. Assume that the first component of the equilibrium point, i.e. $\bar{x}$, is varied according to a cubic polynomial law from the initial value $\bar{x}_0 = 0$ ($t_0 = 3$) to the final value $\bar{x}_1 = 5$ ($t_1 = 15$). The equation of a cubic polynomial as a function of time is

$$q(t) = h(3\tau^2 - 2\tau^3) \text{ with } \tau = \frac{t - t_0}{T} \text{ and } h = q_1 - q_0.$$

   Simulate the behavior of the controlled nonlinear system starting from the initial condition $x_e + \delta x_0$ with $x_e = [\bar{x}_0\ 0\ 0\ 0]^T$ (duration of the simulation 20s) and plot the evolution of $x(t)$ and $\theta(t)$. Superimposed to $x(t)$ plot the signal $\bar{x}(t)$.