

Systems and Control Theory
Master Degree in ELECTRONICS ENGINEERING
 (http://www.dii.unimore.it/~lbiagiotti/SystemsControlTheory.html)

Exercises #5

Matlab start

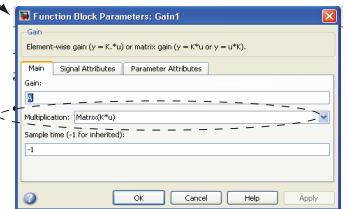
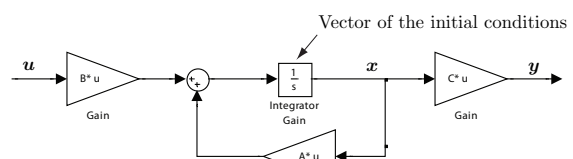
The exercises are carried out under Linux operating system. In order to start the MATLAB program and create the working directory `surname.name`, where all the MATLAB and SIMULINK files must be included, follow the procedure here reported:

1. Login with username and password used for the Unimore e-mail.
2. Open a **Terminal**.
3. Create the working directory and enter it with the commands
`mkdir cognome.nome`
`cd cognome.nome`
4. Open MATLAB with the command `matlab_R2006b`
5. Carry out the exercises, by using M-file, M-functions and Simulink schemes. Remember that the main file must be named `exercise.m` (in the first line of this file specify first name and surname, properly commented).

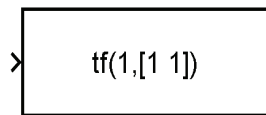
Useful commands and scheme in the Matlab/Simulink environment

continuous-time state-space model definition:	<code>sys = ss(A,B,C,D)</code>
eigenvalues of a matrix:	<code>eig(A)</code>
display of the free evolution:	<code>initial(sys,x0)</code>
display of the forced evolution	
unitary impulse response:	<code>impulse(sys)</code>
unitary step-input response:	<code>step(sys)</code>
saving of data in a vector:	<code>[y,t,x] = initial(sys,x0,tfin)</code> <code>[y,t,x] = impulse(sys,tfin)</code>

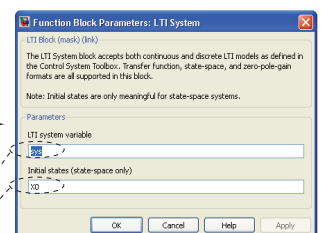
Simulink scheme for the simulation of a state-space model defined by matrices (A,B,C):



LTI block (of the Control System Toolbox) for importing linear, time-invariant (LTI) systems, defined with the commands `tf` or `ss`, into the Simulink environment:



Name of the system
Initial conditions

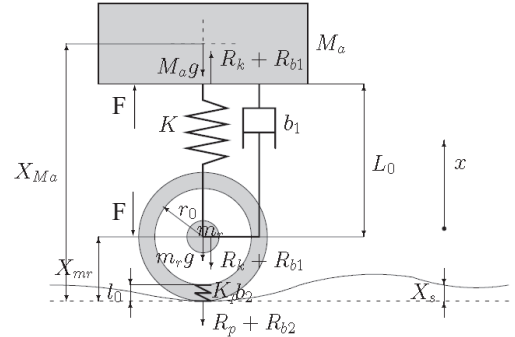


Text of the exercises

Design an M-file (*exercise.m*) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

Consider the mechanical model shown in the figure, representing the simplified model of a car's suspension (same model of Exercises #3). The meaning and the value of the parameters are reported below:

car mass (1/4)	M_a	290	kg
tire mass	m_r	60	kg
stiffness of the suspension	K	20000	N/m
tire stiffness	K_p	190000	N/m
damping of the suspension	b_1	1000	N/m/s
tire damping	b_2	10000	N/m/s



The model of the system in the state-space, with the state vector $\mathbf{x} = [p_a, X_r, p_r, X_{rp}]^T$ (see the POG scheme in the text of Exercises #3 to understand the meaning of the state variables) is

$$\begin{bmatrix} \dot{p}_a \\ \dot{X}_r \\ \dot{p}_r \\ \dot{X}_{rp} \end{bmatrix} = \begin{bmatrix} -b_1/M_a & -K & b_1/m_r & 0 \\ 1/M_a & 0 & -1/m_r & 0 \\ b_1/M_a & K & -(b_1 + b_2)/m_r & -K_p \\ 0 & 0 & 1/m_r & 0 \end{bmatrix} \begin{bmatrix} p_a \\ X_r \\ p_r \\ X_{rp} \end{bmatrix} + \begin{bmatrix} -M_a & 1 & 0 \\ 0 & 0 & 0 \\ -m_r & -1 & b_2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} g \\ F \\ \dot{X}_s \end{bmatrix}$$

and the output variable is $y = \frac{1}{M_a} p_a = \dot{X}_{M_a}$.

1. Define the state-space model by means of the command `ss`.
2. Simulate the free evolution of the system from the initial state $\mathbf{x}_0 = [p_{a,0}, X_{r,0}, p_{r,0}, X_{rp,0}]^T = [0, 0.05, 0, 0]^T$, without using Simulink schemes (duration of the simulation 10 s). Plot in the figure 1 the output \dot{X}_{M_a} (solid blue line). Add grid, axis labels, etc.
3. By using the LTI block of the Control System toolbox, perform the same simulation of previous point by means of a Simulink scheme. In particular, superimpose the system output with the one obtained at point 2 (dashed red line).
4. Design a second Simulink scheme that allows to access to the state of the model (to this purpose adopt the configuration based on a single integrator reported in the previous page). Simulate the system behavior with zero inputs (assume that the gravity acceleration is zero $g = 0$) and initial conditions \mathbf{x}_0 . Plot in the figure 2 (4 distinct subplots) the evolution of all the components of the state vector.
5. Consider zero initial conditions and apply to the system the two constant inputs $g = 9.81 \text{ m/s}^2$ and $F(t) = 0$, while the third input is $\dot{X}_s(t) = h(t-5) - h(t-5.2)$, being $h(t)$ the unit step function (duration of the simulation 10 s). Plot in a new figure the evolution of the output superimposed to the signal $\dot{X}_s(t)$.