Systems and Control Theory

Master Degree Course in ELECTRONICS ENGINEERING

http://www.dii.unimore.it/~lbiagiotti/SystemsControlTheory.html

From regulation to control

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From regulation to the control

- Goal of the regulation: to steer the state vector to the origin (stabilization)

- Question: How a reference command can be tracked?

- Consider the continuous-time system

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad y(t) = Cx(t)$$

with the controller

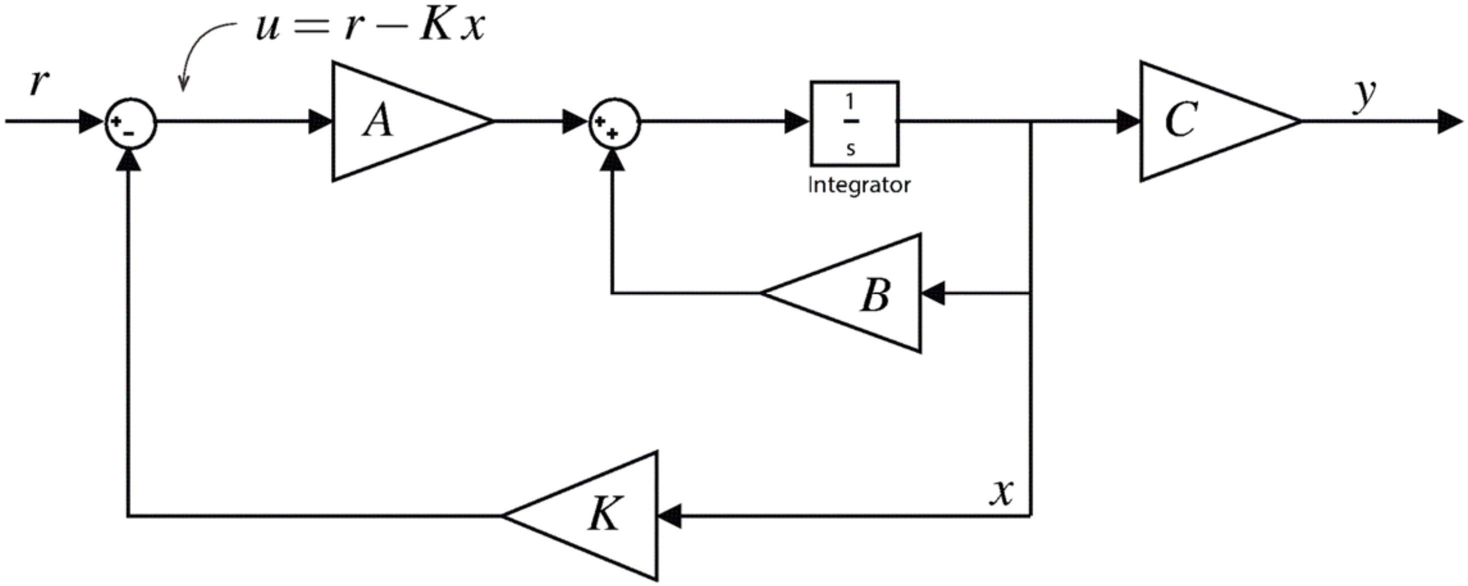
$$u(t) = r(t) - Kx(t)$$

which includes an external command input $r(t)$

- For a good tracking performance

$$y(t) \simeq r(t) \text{ as } t \to \infty$$

From regulation to the control



- For a good tracking performance $y(t) \simeq r(t)$ as $t \to \infty$

- By considering the final value theorem, i.e. $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$
  this performance issue in the frequency domain becomes

$$sY(s) \simeq sR(s) \text{ as } s \to 0 \dashrightarrow \left.\frac{Y(s)}{R(s)}\right|_{s=0} \simeq 1$$ DC gain

From regulation to the control

- The equations of the controlled system in the state-space are

$$\dot{x}(t) = (A - BK)x(t) + Br(t)$$
$$y(t) = Cx(t)$$

leading to

$$\frac{Y(s)}{R(s)} = \underbrace{C\left(sI_n - (A - BK)\right)^{-1}B}_{G_{cl}(s)}$$

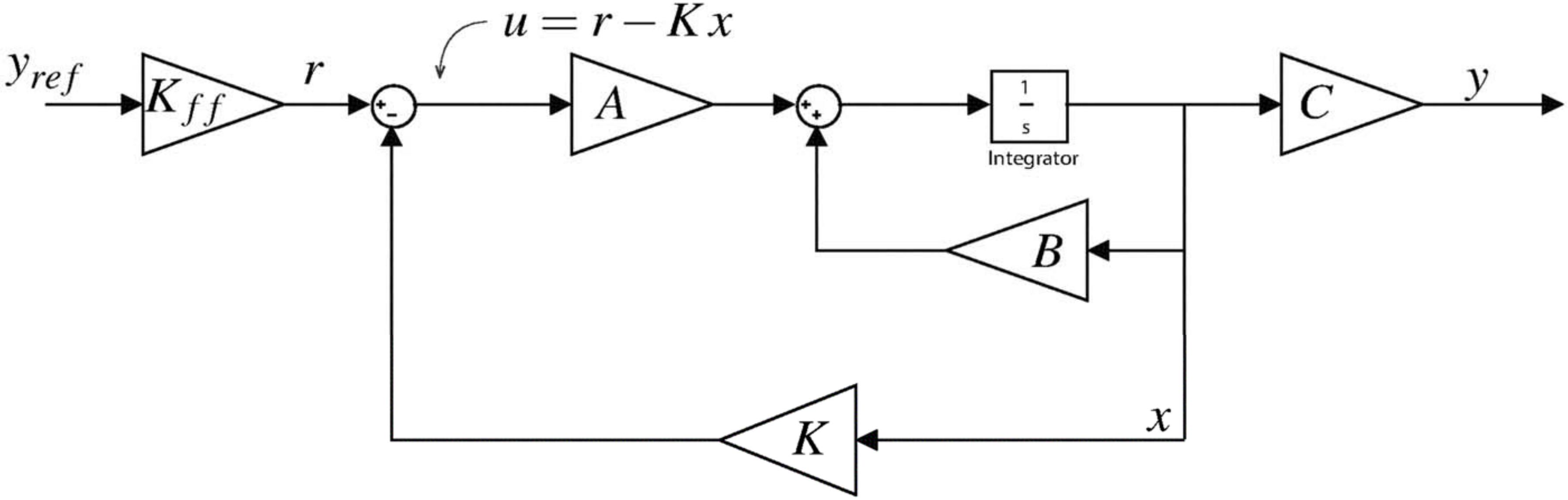
- In general

$$G_{cl}(0) \neq 1$$

From regulation to the control

- A possible solution consists in scaling the reference input by considering an extra gain k_{ff}

- In this way $u(t) = \underbrace{K_{ff} y_{sp}(t)}_{r(t)} - Kx(t)$



From regulation to the control

- The equations of the controlled system with the new input $y_{ref}(t)$ are

$$\dot{x}(t) = (A - BK)x(t) + BK_{ff}y_{ref}(t)$$
$$y(t) = Cx(t)$$

which lead to

$$\frac{Y(s)}{Y_{ref}(s)} = C\left(sI_n - (A - BK)\right)^{-1} BK_{ff} = G_{cl}(s)K_{ff}$$

- By computing

$$K_{ff} = G_{cl}(0)^{-1} = -\left(C(A - BK)^{-1}B\right)^{-1}$$

the dc-gain between $y_{ref}(t)$ and $y(t)$ becomes unitary, i.e.

$$\left.\frac{Y(s)}{Y_{ref}(s)}\right|_{s=0} = 1$$

From regulation to the control: remarks

- This development is based on the assumption that $y_{ref}(t)$ and $r(t)$ are constant. But it can be also used if $y_{ref}(t)$ is a slowly time-varying command

- Analogous considerations hold in the discrete-time domain. In this case, the feed-forward gain K_{ff} should be computed as

$$K_{ff} = -(C(F - GK - I_n)^{-1}G)^{-1}$$

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