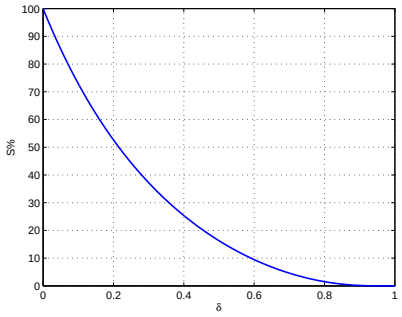
Formule utili per lo svolgimento del compito di sistemi di controllo

- Caratteristiche della risposta di sistemi del secondo ordine:
  - Sorpasso percentuale

$$S\% = 100\,(y_{\max} - 1) = 100\; e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}$$



  - Tempo di assestamento al 5%

$$T_a = \frac{3}{\delta\omega_n}$$

- Formule di inversione per reti anticipatrici $\qquad R_a(s) = \dfrac{1+\tau s}{1+\alpha\tau s}$

$$\tau = \frac{M^\star - \cos\varphi^\star}{\omega_c^\star \sin\varphi^\star}, \qquad \alpha\tau = \frac{\cos\varphi^\star - \frac{1}{M^\star}}{\omega_c^\star \sin\varphi^\star} \qquad \text{con} \qquad \begin{cases} M^\star > 1 \\ 0 < \varphi^\star < 90^o \\ \cos\varphi^\star \;>\; \frac{1}{M^\star} \end{cases}$$

- Formule di inversione per reti ritardatrici $\qquad R_r(s) = \dfrac{1+\alpha\tau s}{1+\tau s}$

$$\alpha\tau = \frac{M^\star - \cos\varphi^\star}{\omega_c^\star \sin\varphi^\star}, \qquad \tau = \frac{\cos\varphi^\star - \frac{1}{M^\star}}{\omega_c^\star \sin\varphi^\star} \qquad \text{con} \qquad \begin{cases} 0 < M^\star < 1 \\ -90^o < \varphi^\star < 0^o \\ M^\star \;<\; \cos\varphi^\star \end{cases}$$

- Filtri di Butterworth $R_{\text{butter}}(s) = \dfrac{1}{B_n(s)}$

ordine	
1	$B_1(s) = s' + 1$
2	$B_2(s) = s'^2 + 1.414s' + 1$
3	$B_3(s) = (s'^2 + s' + 1)(s' + 1)$
4	$B_4(s) = (s'^2 + 0.765s' + 1)(s'^2 + 1.848s' + 1)$

- Tuning dei regolatori PID
  - Tabelle per il tuning in anello aperto

Contr.	Ziegler-Nichols	Cohen-Coon	3C
P	$\mu K_p = (T/\tau)^{-1}$	$\mu K_p = (T/\tau)^{-1} + 0.33$	$\mu K_p = 1.2(T/\tau)^{-.956}$
PI	$\mu K_p = 0.9(T/\tau)^{-1}$	$\mu K_p = 0.9(T/\tau)^{-1} + 0.082$	$\mu K_p = 0.93(T/\tau)^{-.956}$
	$T_i/\tau = 3.33(T/\tau)$	$T_i/\tau = \dfrac{3.33(T/\tau)[1 + (T/\tau)/11]}{1 + 2.2(T/\tau)}$	$T_i/\tau = 0.93(T/\tau)^{.583}$
PID	$\mu K_p = 1.2(T/\tau)^{-1}$	$\mu K_p = 1.35(T/\tau)^{-1} + 0.27$	$\mu K_p = 1.37(T/\tau)^{-.95}$
	$T_i/\tau = 2(T/\tau)$	$T_i/\tau = \dfrac{2.5(T/\tau)[1 + (T/\tau)/5]}{1 + 0.6(T/\tau)}$	$T_i/\tau = 0.74(T/\tau)^{.738}$
	$T_d/\tau = 0.5(T/\tau)$	$T_d/\tau = \dfrac{0.37(T/\tau)}{1 + 0.2(T/\tau)}$	$T_d/\tau = 0.365(T/\tau)^{.95}$

Criterio	Controllore	K_p	T_i	T_d
IAE	PI	$\dfrac{0.758}{\mu}(T/\tau)^{-0.861}$	$\dfrac{\tau}{1.02 - 0.323(T/\tau)}$	
ITAE	PI	$\dfrac{0.586}{\mu}(T/\tau)^{-0.916}$	$\dfrac{\tau}{1.03 - 0.165(T/\tau)}$	
IAE	PID	$\dfrac{1.086}{\mu}(T/\tau)^{-0.869}$	$\dfrac{\tau}{0.74 - 0.13(T/\tau)}$	$\tau\ 0.348(T/\tau)^{0.914}$
ITAE	PID	$\dfrac{0.965}{\mu}(T/\tau)^{-0.855}$	$\dfrac{\tau}{0.796 - 0.147(T/\tau)}$	$\tau\ 0.308(T/\tau)^{0.929}$

  - Tabelle per il tuning in anello chiuso

Controllore	Parametri
PI	$K_p = 0.45k^\star$
	$T_i = 0.85T^\star$
PD	$K_p = 0.5k^\star$
	$T_d = 0.2T^\star$
PID	$K_p = 0.6k^\star$
	$T_i = 0.5T^\star$
	$T_d = 0.12T^\star$

- Pianificazione delle traiettorie
  - Principali traiettorie (espresse in forma normalizzata) e relativi valori limite
    - Traiettoria polinomiale di grado 3

$$\begin{aligned} q_N(\tau) &= 3\tau^2 - 2\tau^3 \\ q_N^{(1)}(\tau) &= 6\tau - 6\tau^2 \\ q_N^{(2)}(\tau) &= 6 - 12\tau \\ q_N^{(3)}(\tau) &= -12. \end{aligned}$$

$$q_{N\,max}^{(1)} = q_N^{(1)}(0.5) = \frac{3}{2}$$

$$q_{N\,max}^{(2)} = q_N^{(2)}(0) = 6$$

    - Traiettoria polinomiale di grado 5

$$
\begin{aligned}
q_N(\tau) &= 10\tau^3 - 15\tau^4 + 6\tau^5 \\
q_N^{(1)}(\tau) &= 30\tau^2 - 60\tau^3 + 30\tau^4 \\
q_N^{(2)}(\tau) &= 60\tau - 180\tau^2 + 120\tau^3 \\
q_N^{(3)}(\tau) &= 60 - 360\tau + 360\tau^2
\end{aligned}
\qquad
\begin{aligned}
q_{N\,max}^{(1)} &= q_N^{(1)}(0.5) = \frac{15}{8} \\
q_{N\,max}^{(2)} &= q_N^{(2)}(0.2123) = \frac{10\sqrt{3}}{3} \\
q_{N\,max}^{(3)} &= q_N^{(3)}(0) = 60
\end{aligned}
$$

∗ Traiettoria cicloidale

$$
\begin{aligned}
q_N(\tau) &= \tau - \tfrac{1}{2\pi}\sin 2\pi\tau \\
q_N^{(1)}(\tau) &= 1 - \cos 2\pi\tau \\
q_N^{(2)}(\tau) &= 2\pi \sin 2\pi\tau \\
q_N^{(3)}(\tau) &= 4\pi^2 \cos 2\pi\tau
\end{aligned}
\qquad
\begin{aligned}
q_{N\,max}^{(1)} &= q_N^{(1)}(0.5) = 2 \\
q_{N\,max}^{(2)} &= q_N^{(2)}(0.25) = 2\pi \\
q_{N\,max}^{(3)} &= q_N^{(3)}(0) = 4\pi^2
\end{aligned}
$$

∗ Traiettoria armonica

$$
\begin{aligned}
q_N(\tau) &= \tfrac{1}{2}(1 - \cos \pi\tau) \\
q_N^{(1)}(\tau) &= \tfrac{\pi}{2}\sin \pi\tau \\
q_N^{(2)}(\tau) &= \tfrac{\pi^2}{2}\cos \pi\tau \\
q_N^{(3)}(\tau) &= -\tfrac{\pi^3}{2}\sin \pi\tau.
\end{aligned}
\qquad
\begin{aligned}
q_{N\,max}^{(1)} &= q_N^{(1)}(0.5) = \tfrac{\pi}{2} \\
q_{N\,max}^{(2)} &= q_N^{(2)}(0) = \tfrac{\pi^2}{2} \\
|q_N^{(3)}|_{max} &= |q_N^{(3)}(0.5)| = \tfrac{\pi^3}{2}
\end{aligned}
$$

– Traiettoria trapezoidale con $\mathtt{a}_{max}$ e $\mathtt{v}_{max}$ assegnate:

$$
\begin{cases}
T_a = \dfrac{\mathtt{v}_{max}}{\mathtt{a}_{max}} \\
T = \dfrac{h}{\mathtt{v}_{max}} + \dfrac{\mathtt{v}_{max}}{\mathtt{a}_{max}}
\end{cases}
\qquad \text{se } T_a \le \frac{T}{2}
$$

$$
\begin{cases}
T_a = \sqrt{\dfrac{h}{\mathtt{a}_{max}}} \\
T = 2T_a \\
\dot{q}_{max} = \mathtt{a}_{max} T_a = \sqrt{\mathtt{a}_{max} h} = \dfrac{h}{T_a}
\end{cases}
\qquad \text{altrimenti}
$$

Traiettoria trapezoidale con T e T_a assegnate:

$$
\begin{cases}
\mathtt{v}_{max} = \dfrac{h}{T - T_a} \\
\mathtt{a}_{max} = \dfrac{h}{T_a(T - T_a)}.
\end{cases}
$$

$$
q(t) = \begin{cases}
q_0 + \dfrac{1}{2}\mathtt{a}_{max}(t - t_0)^2, & t_0 \le t \le t_0 + T_a \\
q_0 + \mathtt{a}_{max} T_a \left(t - t_0 - \dfrac{T_a}{2}\right), & t_0 + T_a < t \le t_1 - T_a \\
q_1 - \dfrac{1}{2}\mathtt{a}_{max}(t_1 - t)^2, & t_1 - T_a < t \le t_1.
\end{cases}
$$