Systems and Control Theory

Master Degree in ELECTRONICS ENGINEERING

(http://www.dii.unimore.it/∼lbiagiotti/SCT.html)

Exercises #5

Useful commands and scheme in the Matlab/Simulink environment

With reference to the system

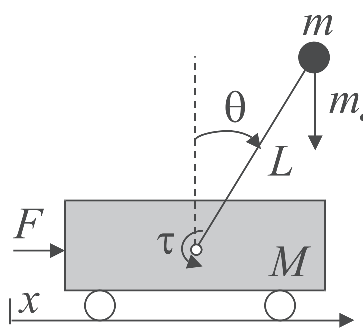
$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \end{cases}$$

Controllability matrix	`CO = ctrb(A,B)`
Observability matrix	`OB = obsv(A,C)`
Rank of a marix $\boldsymbol{M}$	`rank(M)`
Eigenvalues of a matrix $\boldsymbol{M}$	`E = eig(M)`
Computation the feedback control law $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x}$ for single input systems	`K = acker(A,B,P)` where `P` is the vector of the desired eigenvlues
Computation the feedback control law $\boldsymbol{u} = -\boldsymbol{K}\boldsymbol{x}$ (**N.B.** any number of inputs but no eigenvalue should have a multiplicity greater than the number of inputs)	`K = place(A,B,P)` where `P` is the vector of the desired eigenvlues
With reference to the asymptotic estimator $\dot{\hat{\boldsymbol{x}}}(\boldsymbol{t}) = \boldsymbol{A}\hat{\boldsymbol{x}}(\boldsymbol{t}) + \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t}) + \boldsymbol{L}(\boldsymbol{y}(\boldsymbol{t}) - \hat{\boldsymbol{y}}(\boldsymbol{t}))$, computation of the gain $\boldsymbol{L}$	`L = acker(A',C',Po)'` where `Po` is the vector of the desired eigenvlues (single output systems)
	`L = place(A',C',Po)'` where `Po` is the vector of the desired eigenvlues, whose multiplicity should be not greater than the number of outputs
Solution of the continuous-time algebraic Riccati equation	`[S,E,K] = care(A,B,Q,R)` where `S` is the solution of the equation, `E` are thh closed-loop eigenvalues and `K` is the Kalman gain ($u(t) = -kx(t)$)
Solution of the continuous-time LQ optimal control problem	`[K,S,E] = lqr(A,B,Q,R)` where `S` is the solution of the equation, `E` are thh closed-loop eigenvalues and `K` is the Kalman gain ($u(t) = -kx(t)$)

Text of the exercises

Design an M-file (exercise.m) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

Let's consider the cart-pole system shown in the figure. By assuming that the mass of the cart is M and the mass m of the inverted pendulum is lumped at the end of massless rigid rod, the model of the system is:



$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \dfrac{m\,L\,x_4^2\,\sin(x_3) - m\,g\,\sin(x_3)\,\cos(x_3) + u}{M + m\,(\sin(x_3))^2} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \dfrac{-m\,L\,x_4^2\,\sin(x_3)\cos(x_3) + (M+m)\,g\,\sin(x_3) - u\,\cos(x_3)}{L\,\left(M + m\,(\sin(x_3))^2\right)} \end{cases}$$

where the state variables are

$$\mathbf{x} = [\,x_1 \;\; x_2 \;\; x_3 \;\; x_4\,]^T = [\,x \;\; \dot{x} \;\; \theta \;\; \dot{\theta}\,]^T$$

and the input $u = F$ is the force applied to the cart.

1. By considering the numerical values $m = 0.1$ kg, $M = 1$ kg, $L = 1$ m, $g = 9.81$ m/s^2, build the Simulink model that allows to simulate the dynamics of the nonlinear system. Assume that the measured variables are the cart position x and the pendulum angular position θ, i.e. $y = [x,\, \theta]^T$.

2. Linearize the system about the equilibrium point $x_e = [\bar{x}\ 0\ 0\ 0]^T$, where $\bar{x}$ is a generic cart position (in the Simulink model assume $\bar{x} = 1$), corresponding to the input $u_e = 0$ and analyze its stability.

3. Design a LQ regulator for the linearized system that minimizes the performance index

$$J = \frac{1}{2}\int_0^\infty 5\delta x^2(t) + \delta\theta^2(t) + \delta u^2(t)\,dt \tag{1}$$

   Simulate the evolution of the controlled (linear) system from the initial conditions $\delta x_0 = [0\ 0\ 0.55\ 0]^T$ (duration of the simulation 6s). Plot in the same figure (2 subplots) the evolution of $\delta x(t)$ (1st subplot) and $\delta\theta(t)$ (2nd subplot).

4. After having analyzed the observability of the system, design an asymptotic state estimator and insert it in the Simulink scheme. Plot in a new figure the evolution of $\delta x(t)$ and $\delta\theta(t)$ obtained with the same initial conditions of previous point and in a new figure compare the components of the actual state and those of the estimated state.

5. Apply the dynamic output feedback to the nonlinear system and simulate the evolution from the initial condition $x_e + \delta x_0$. Plot in a new figure the evolution of $x(t)$ and $\theta(t)$.

6. Assume that the first component of the equilibrium point, i.e. $\bar{x}$, is varied according to a cubic polynomial law from the initial value $\bar{x}_0 = 0$ ($t_0 = 3$) to the final value $\bar{x}_1 = 5$ ($t_1 = 15$). The equation of a cubic polynomial as a function of time is

$$q(t) = h(3\tau^2 - 2\tau^3) \text{ with } \tau = \frac{t - t_0}{T} \text{ and } h = q_1 - q_0.$$

   Simulate the behavior of the controlled nonlinear system starting from the initial condition $x_e + \delta x_0$ with $x_e = [\bar{x}_0\ 0\ 0\ 0]^T$ (duration of the simulation 20s) and plot the evolution of $x(t)$ and $\theta(t)$. Superimposed to $x(t)$ plot the signal $\bar{x}(t)$.