Systems and Control Theory

Master Degree in ELECTRONICS ENGINEERING

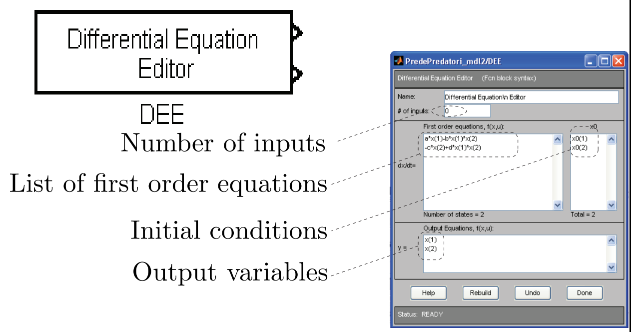
(http://www.dii.unimore.it/~lbiagiotti/SCT.html)

Exercises #4

Useful commands and scheme in the Matlab/Simulink environment

Block for importing differential equations in the Simulink environment:

`dee` at the Matlab prompt



Input-output ports for specifying the desired inputs and outputs (and consequently the matrices B and C of the linearized system):

Command for finding the equilibrium points of a dynamic system:

`[xe,ue,ye,dx]=trim('sys_mdl',x0,u0,y0,ix,iu,iy)` where `sys_mdl` is the name of the Simulink model to be linearized and `x0`, `u0`, `y0` are the initial guesses of state, input and output from which the equilibrium values are sought, `ix`, `iu` and `iy` are the vectors of indices that specify which components of `xe`, `ue`, `ye` must be equal to the values given in `x0`, `u0`, `y0`. *Example.* The equilibrium point of a third order system with only one input, for `u=u*`, sought starting from `x0=[0 0 0]'`: `[xe,ue,ye,dx]=trim('sys_mdl',[0 0 0]',u*,[],[],[1],[])`

Command for the linearization of a Simulink model `sys_mdl`:

[A,B,C,D]=linmod('sys_mdl',xe,ue)

Command for determining the position of the state variables of a POG model (integrators'outputs) inside the state vector of the linearized system

`[sizes,x0,xstring] = <Simulink Filename>` where`<Simulink Filname>` is the name of the Simulink file (without quotes and file extension) which contains the nonlinear POG model. See the content of the variable `xstring`.

Text of the exercises

Design an M-file (exercise.m) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

a) the system of nonlinear differential equations

$$\begin{cases} \dot{x}_1 &= a\,x_1 - b\,x_1\,x_2 \\ \dot{x}_2 &= -c\,x_2 + d\,x_1\,x_2 \end{cases} \tag{1}$$

represents the interplay between two population living in the same ecosystem. The population x_1 (prey) breeds in an exponential way but it is subject to predation by the polpulation x_2 (predators). On the other hand, the population x_2 decreases exponentially and can only grows as a consequence of the predation. The meaning of the parameters is:

- $a = 0.9$ is the reproductive power of prey;
- $b = 0.01$ determines the decrease of prey due to predation;
- $c = 0.8$ determines the reduction of the predators' population due to the lack of prey;
- $d = 0.01$ is the benefit of the hunting for the predators.

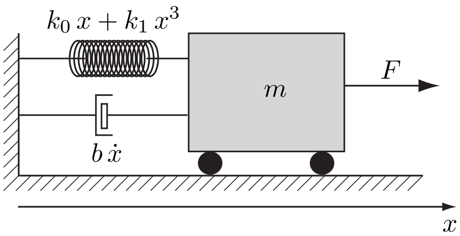
With reference to equation (1):

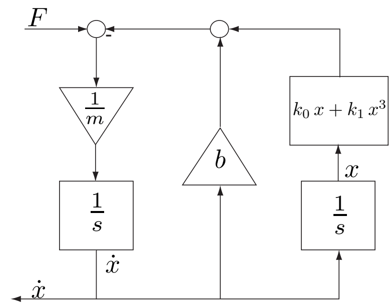
1. Simulate, by means of a Simulink scheme, the dynamics of the two populations from the initial conditions $x_1(0) = 70$, $x_2(0) = 70$ (note that the system has no input); duration of the simulation 100 s. Plot in the $x_1 - x_2$ plane the state trajectories and in a new figure the evolution of prey and predators as a function of time.
2. Linearize the system about the equilibrium point $(x_{1,e},\, x_{2,e}) = \left(\frac{c}{d},\, \frac{a}{b}\right)$. Analyze the stability of the nonlinear system by taking into account the eigenvalues of the matrix A.
3. Compare the response of the nonlinear system and of the linearized system from the same initial conditions considered at point 1. Note that the linearized model describes the dynamics of the system with respect to the equilibrium point, that is

$$\begin{aligned} \delta x &= x - x_e \\ \delta u &= u - u_e \\ \delta y &= y - y_e \end{aligned}$$

where $(\delta x,\, \delta u,\, \delta y)$ are the variables of the linearized system and $(x,\, u,\, y)$ the variables of the nonlinear system. As a consequence, for a given u the equivalent input applied to the linearized system is $\delta u = u - u_e$ (or conversely, given δu, u is $u_e + \delta u$). The output δy of the linearized system must be add to the equilibrium output y_e in order to find the (approximated) output y of the nonlinear system. Compare the state trajectories of the nonlinear system and those of the linear system. In a new figure, plot the evolution of x_1 and x_2 (2 subplots) in the two cases as a function of time.

b) Given the mechanical system reported in the figure (mass-nonlinear spring system) and the corresponding POG scheme





where $u = F$ represents the input of the system, $y = \dot{x}$ is the output and the parameters'values are $m = 1$ Kg, $k_0 = 0.5$ N/m $k_1 = 0.25$ N/m^3, $b = 0.75$ M s/m:

1. Build the Simulink model of the nonlinear system.
2. Linearize the model about the equilibrium point $(\dot{x}_e,\, x_e)$ obtained with the constant input $F_e = 3$ N.
3. Compare the response of the nonlinear system and of the linearized system from the initial conditions $(\dot{x}(0),\, x(0)) = (\dot{x}_e,\, x_e)$ and applying the input $u(t) = F_e + h(t-1)$ where $h(t)$ is the unit step function (duration of the simulation 10 s). Plot the output of the two models as a function of time.