Systems and Control Theory

Master Degree in ELECTRONICS ENGINEERING

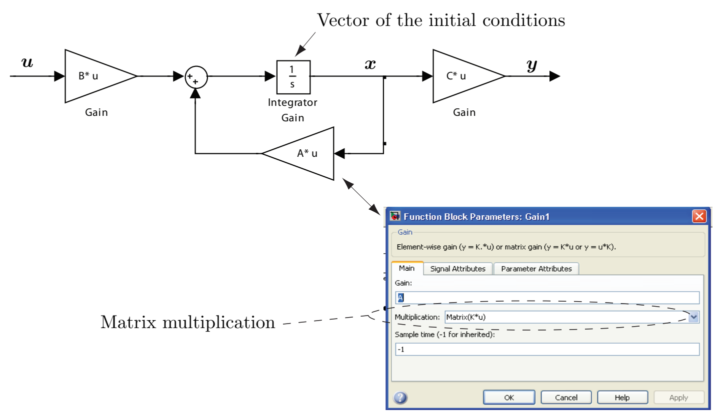
(http://www.dii.unimore.it/~lbiagiotti/SCT.html)

Exercises #3

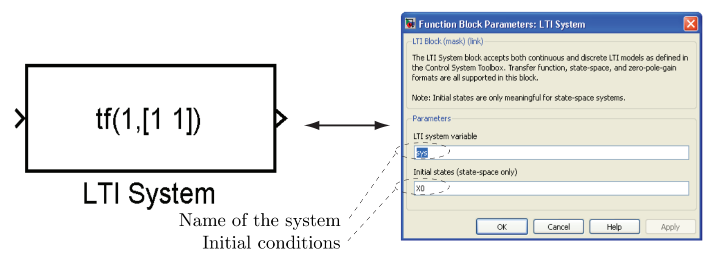
Useful commands and scheme in the Matlab/Simulink environment

continuous-time state-space model definition:	`sys = ss(A,B,C,D)`
eigenvalues of a matrix:	`eig(A)`
display of the free evolution:	`initial(sys,x0)`
display of the forced evolution	
unitary impulse response:	`impulse(sys)`
unitary step-input response:	`step(sys)`
saving of data in a vector:	`[y,t,x] = initial(sys,x0,tfin)` `[y,t,x] = impulse(sys,tfin)`

Simulink scheme for the simulation of a state-space model defined by matrices `(A,B,C)`:



LTI block (of the Control System Toolbox) for importing linear, time-invariant (LTI) systems, defined with the commands `tf` or `ss`, into the Simulink environment:

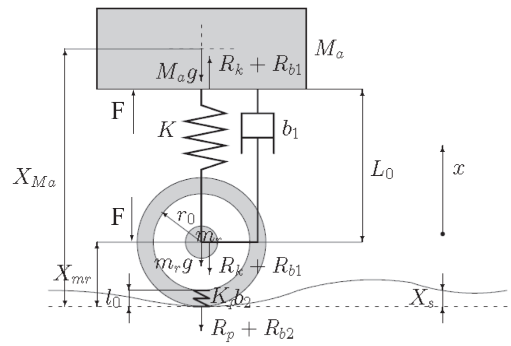


Text of the exercises

Design an M-file (exercise.m) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

Consider the mechanical model shown in the figure, representing the simplified model of a car's suspension (same model of Exercises #3). The meaning and the value of the parameters are reported below:

car mass (1/4)	M_a	290	kg
tire mass	m_r	60	kg
stiffness of the suspension	K	20000	N/m
tire stiffness	K_p	190000	N/m
damping of the suspension	b_1	1000	N/m/s
tire damping	b_2	10000	N/m/s



The model of the system in the state-space, with the state vector $\boldsymbol{x} = [p_a,\, X_r,\, p_r,\, X_{rp}]^T$ (see the POG scheme in the text of Exercises #3 to understand the meaning of the state variables) is

$$\begin{bmatrix} \dot{p}_a \\ \dot{X}_r \\ \dot{p}_r \\ \dot{X}_{rp} \end{bmatrix} = \begin{bmatrix} -b_1/M_a & -K & b_1/m_r & 0 \\ 1/M_a & 0 & -1/m_r & 0 \\ b_1/M_a & K & -(b_1+b_2)/m_r & -K_p \\ 0 & 0 & 1/m_r & 0 \end{bmatrix} \begin{bmatrix} p_a \\ X_r \\ p_r \\ X_{rp} \end{bmatrix} + \begin{bmatrix} -M_a & 1 & 0 \\ 0 & 0 & 0 \\ -m_r & -1 & b_2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} g \\ F \\ \dot{X}_s \end{bmatrix}$$

and the output variable is $y = \frac{1}{M_a} p_a = \dot{X}_{M_a}$.

1. Define the state-space model by means of the command `ss`.
2. Simulate the free evolution of the system from the initial state $\boldsymbol{x}_0 = [p_{a,0},\, X_{r,0},\, p_{r,0},\, X_{rp,0}]^T = [0,\, 0.05,\, 0,\, 0]^T$, without using Simulink schemes (duration of the simulation 10 s). Plot in the figure 1 the output $\dot{X}_{M_a}$ (solid blue line). Add grid, axis labels , etc.
3. By using the LTI block of the Control System toolbox, perform the same simulation of previous point by means of a Simulink scheme. In particular, superimpose the system output with the one obtained at point 2 (dashed red line).
4. Design a second Simulink scheme that allows to access to the state if the model (to this purpose adopt the configuration based on a single integrator reported in the previous page). Simulate the system behavior with zero inputs (assume that the gravity acceleration is zero $g = 0$) and initial conditions $\boldsymbol{x}_0$. Plot in the figure 2 (4 distinct subplots) the evolution of all the components of the state vector.
5. Consider zero initial conditions and apply to the system the two constant inputs $g = 9.81 m/s^2$ and $F(t) = 0$, while the third input is $\dot{X}_s(t) = h(t-5) - h(t-5.2)$, being $h(t)$ the unit step function (duration of the simulation 10 s). Plot in a new figure the evolution of the output superimposed to the signal $\dot{X}_s(t)$.