Systems and Control Theory
Master Degree in ELECTRONICS ENGINEERING

(http://www.dii.unimore.it/~lbiagiotti/SCT.html)

Text of the exercises

Design an M-file (exercise.m) that, with the help of other M-files and SIMULINK schemes if necessary, solves the following problems.

1. Define the MATLAB function `[x] = LinearSystem(A,b)`, that finds the solution $\mathbf{x}$ of a generic system of linear equations, $\mathbf{A}\mathbf{x} = \mathbf{b}$. Use the function to solve the system

$$\begin{cases} x_1 + x_2 + x_3 - x_4 = 1 \\ x_1 + x_2 - x_3 = 2 \\ x_1 - x_2 + x_3 = 0 \\ x_1 + 2x_2 - 3x_3 = 2 \end{cases}$$

2. Define the MATLAB function

`[A,B,C,D] = ControllableCanonicalForm(Num,Den)`

that, starting from the transfer function of a SISO system (`Num` and `Den` are the vectors of the coefficients of the numerator and denominator polynomials, respectively), provides the matrices of the state-space representation of the system in the controllable canonical form. Note that the relation between the n-th order transfer function

$$G(s) = \frac{c_{n-1}s^{n-1} + c_{n-2}s^{n-2} + \ldots + c_1 s + c_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0}$$

and the matrices of the system is

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -a_0 & -a_1 & -a_2 & \ldots & -a_{n-1} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = [c_0,\ c_1,\ \ldots,\ c_{n-2},\ c_{n-1}], \qquad \mathbf{D} = 0.$$

Hint: the number of zeros of the system may be smaller than $n-1$. As a consequence, some of the coefficients c_i may be null.
Useful commands are `eye(n)`, `zeros(n,m)`, `fliplr(X)`. *For their use refer to the help.*

Given the system $G(s) = \dfrac{10s + 10}{s^3 - 1.6s^2 - 15.4s + 6.1}$ find its model in the state-space representation by using the newly defined function.

3. Define the MATLAB function `[q_t] = TrjPoly3(q0,q1,T,dt)`, that returns a vector containing the samples (computed with time-step `dt`) of a third-order polynomial trajectory from the initial point q_0 to the final point q_1 in a duration T. Note that the analytical equation of the trajectory is

$$q(t) = q_0 + h\left(3\left(\frac{t}{T}\right)^2 - 2\left(\frac{t}{T}\right)^3\right), \qquad 0 \le t \le T,$$

where $h = q_1 - q_0$. With the new function compute a trajectory from $q_0 = 0$ to $q_1 = 3$ ($T = 2$) and plot its behavior as a function of time.