

Different Graphical Descriptions of Clutch Modelling for Traction Systems

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Abstract – In this paper the friction clutch of a vehicle is modelled using three special graphical tools which are developed to model power flow through physical systems: energetic macroscopic representation, power oriented graph and bond graph. Due to its non-linear behaviour, the modelling of the clutch is a challenging point. Indeed, it connects or disconnects mechanical parts, which can have similar or diverging rotation speeds. The aim of the paper is to highlight the features of each tool for the clutch modelling, the simulation and the analysis of such systems including non-linearities.

I. INTRODUCTION

Vehicles (thermal, electrical as well as hybrid) are complex systems because they are composed of several devices of different nature (continuous, discrete, linear, non-linear...) and of different fields of physics. In order to compare and optimize vehicles (in particular for the hybrid vehicles), global simulations are required taking into account the main components and their interactions. Specific modelling has thus been suggested to simulate the whole system [1]-[4]. Because of its non-linear behaviour, the modelling of friction clutch is a sensitive point.

Friction influences inside of mechanical systems have been the subject of many publications [5], [6] and there are several approaches employed to model a clutch according to the literature. Two of these methods are based on the Lagrange operators using reduced matrices and state space formulations. These models are thus composed of complex mathematical equations [7], [8]. Two other methods, Bond Graph (BG) [9] and Power Oriented Graph (POG) [3], [10], define that the clutch can be represented by the same number of equations for the two states of a friction clutch: slipping or locked. In this case there is no switching necessary because these methods express the torque delivered by the clutch at each point of time [8]. The drawback of this modelling is the necessity of the consideration of two inertias which are associated to the both clutch states. Please note, if the clutch is closed the plates have the same rotation speed and there is a single equivalent inertia only. As well as POG and BG are used in this paper to generate friction clutch models without explicit switching between partial models but with attention to the power flow.

An object oriented non-causal language like Modelica would model the clutch by taking into account the two mentioned states [4], [11]. As each state is represented by its own model, this method requires a switching between these alternating models using finite automata or Petri nets [12]. However, due to the fact that Modelica may produce non-causal models,

the method of Energetic Macroscopic Representation (EMR) is used in this paper to model such a non-linear device via explicit switching between two partial models and attention to the modelling of the power flow [13]-[15].

After exposition of the studied object, the models of a single clutch based on EMR, POG and BG are presented and discussed. Finally simulation results via Simulink and comparisons are given.

II. THE STUDIED FRICTION CLUTCH

In the case of a conventional vehicle the clutch allows the power to be transmitted from the engine to the wheels in order to change the speed ratio using the gearbox. This is done by an external actuator not considered in this paper. In principle the clutch is non-linear and changes between two states. Taking into consideration as proper assumptions the neglect as well as of temperature and abrasion effects and of the vibration dampers in addition the behaviour may be described by the following relationships whereat p_{clutch} represents the position (in percent) of the clutch release bearing (Fig. 1). The behaviour of the clutch is non-linear and two different states have to be considered. When the clutch is slipping (or is open) two inertias must be taken into account when considering the system. However, when it is locked, the system must be considered as a single equivalent inertia:

Slipping clutch – When the clutch slips or is open the clutch differential relationships can be expressed as:

$$J_1 \frac{d}{dt} \omega_1 + f_1 \omega_1 = T_1 - T_2 \quad (1)$$

$$J_2 \frac{d}{dt} \omega_2 + f_2 \omega_2 = T_3 - T_4 \quad (2)$$

$$T_2 = T_3 = 2 F_n R_{clutch} \mu \text{sign}(\omega_1 - \omega_2) \quad (3)$$

$$F_n = F_{max} \left[I - \sqrt{I - \left(\frac{p_{clutch} - I}{100} \right)^2} \right] \quad (4)$$

$$\mu = \mu_{stat} - \alpha_{dyn} |\omega_1 - \omega_2| \quad (5)$$

where ω_1 and ω_2 are the speeds of the primary and secondary clutch; T_1 and T_4 are the torques provided upstream and downstream of the transmission respectively; T_2 and T_3 are the torques produced by the clutch; f_1, f_2 and J_1, J_2 are the friction

coefficients and the inertias of the shafts respectively; F_n is the normal actuation force; R_{clutch} is the average radius of the clutch friction plate; μ is the total friction coefficient with μ_{stat} and μ_{dyn} respectively the static friction coefficient and the dynamic friction coefficient of the clutch.

Locked clutch – When the clutch is locked, the friction plate is rigidly coupled with the primary and secondary plates. Both differential relationships (1) and (2) can no longer be independent because $\omega_1 = \omega_2$. The following relationships are obtained:

$$J_{lock} \frac{d}{dt} \omega_2 + f_{lock} \omega_2 = T_1 - T_4 \quad (6)$$

$$\begin{cases} f_{lock} = f_1 + f_2 \\ J_{lock} = J_1 + J_2 \end{cases} \quad (7)$$

$$\begin{cases} T_{lock} = T_1 \\ \omega_1 = \omega_2 \end{cases} \quad (8)$$

where f_{lock} and J_{lock} are the friction coefficient and the inertia moment of the equivalent shaft respectively.

III. MODELLING BASED ON ENERGETIC MACROSCOPIC REPRESENTATION

EMR is based on the principle of action and reaction between connected elements and defines especially geometrical icons (see Appendix) regarding the energy domain and the handling of the power whereat energy storage, even as integral causality required, and losses are typically subsumed in one element [14], [15]. To simulate the introduced clutch two models are used for EMR: one for the clutch slipping (Fig. 2), another for the clutch locked (Fig. 3). Both models are connected using a switch selector in order to respect the criteria necessary to ensure the physical energy flow required during commutation between both models. The commutation may be performed by a Petri net – compare Fig. 5.

A postulation of same angular accelerations on both sides of the clutch yields to derivation of relationship (11). Each component of EMR models can be internally described using transfer functions or state models for instance. The power exchanged between two elements is the scalar product of the associated exchange vectors. For example, the product of the action vector (torque T_1) and the reaction vector (speed ω_1) of the Mechanical Source 1 (MS1) gives the power $P_1 = T_1 \omega_1$. The exchange vectors themselves are associated in pairs always: each action vector corresponds to a reaction vector.

Both models are depicted by EMR with two switch selectors (Fig. 4). To activate the first or the second model, the switch selectors are defined using a Petri net (Fig. 5) [12]. The states of the Petri net represent the active model. When the clutch slips, the state P_1 is active: the model 1 is used. The state P_2 is activated if the transition τ_1 is true: the clutch is locked when primary and secondary speeds are equal. When the clutch is considered locked, the state P_2 is active: the model 2 is used. The state P_1 is activated if the transition τ_2 is true: the clutch

slips when the absolute value of the clutch torque $|T_2|$ (9) are greater than the maximal torque T_{max} (10). The clutch torque T_2 is determined from the relationships (1), (2) and (8) [13]:

$$T_2 = T_3 = \frac{J_2 T_1 + J_1 T_4 + (J_1 f_2 - f_1 J_2) \omega_2}{J_{lock}} \quad (9)$$

$$T_{max} = 2 F_n R_{clutch} \mu = 2 F_n R_{clutch} \mu_{stat} \quad (10)$$

Moreover the EMR modelling procedure uses a special method in addition to above mentioned ones as a possibility to describe the internally function of the basic subsystems (elements) as well as to locate a naturally, sensible control structure, i.e. to locate controllers and measurements or estimations / observers. This is possible because EMR is principally based on an integral causality on the one hand and it clearly indicates the location of energy storages on the other hand. Moreover inversions of coupling elements require power distribution or weighted inputs. These inputs lead to a strictly logical organization of the energy distribution using EMR.

IV. MODELLING BASED ON POWER ORIENTED GRAPH

A POG explicitly defines visible forward and backward directions of a connection between two elements as valid for EMR too [16], [17]. As known for transfer functions the Laplace domain is used typically and all mathematical operations can be immediately and clearly seen in contrast to BG and EMR.

One can state a close relationship to the BG method because both are based on the same idea of “power interaction” between subsystems as a key element for the modelling. The principal difference with Bond Graph (see below) is the explicit differentiation between the flow and effort variables. The physical and mathematical equivalences (like the state model) are easily deductible.

But a POG only uses two types of blocks: elaboration (energy storage / dissipation – see Appendix) and connection (energy transformation). Elaboration blocks include as well as summarization of storage and losses or not of a non-ideal storage element and the optional “output” of displacement / momentum if integration and parameter multiplication are separated modelled. Four-sided figures symbolize elaboration and connection whereas a circle above or below an elaboration block represents a summation element. Moreover if a black spot is present near to an entering arrow, the corresponding variable must get a negative sign before the sum operation. The block schemes obtained with this modelling technique are simple, modular and easy to use – compare Fig. 6, Fig. 7 and relationships (1), (2). POG and BG may use an exactly same subsystem NL, therefore please see section “BG” below for details.

V. MODELLING BASED ON BOND GRAPH

Bond Graph defines common bidirectional connections marked by a half arrow (preferred energy transmission direction) and a short bar at one side (causality) [18]-[20]. Similar to

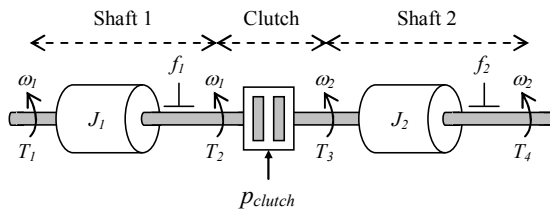


Fig. 1. Transmission scheme

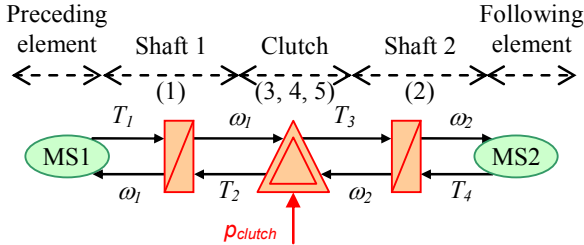


Fig. 2. EMR of the clutch in slipping condition (Model 1)

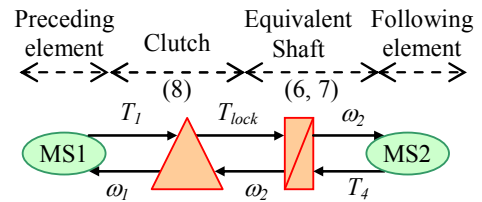


Fig. 3. EMR of the clutch in locked condition (Model 2)

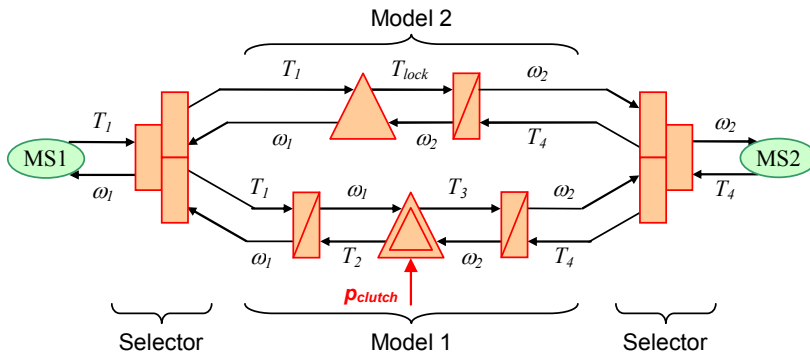


Fig. 4. EMR of the transmission with clutch

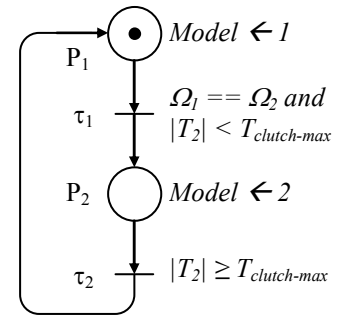


Fig. 5. Petri net of switch selector

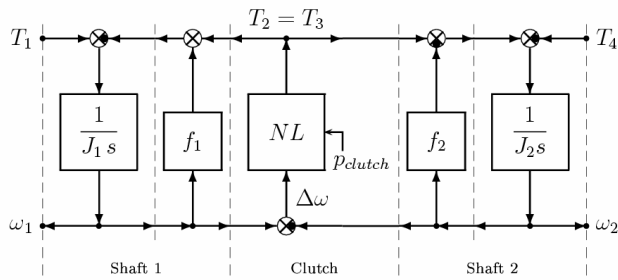


Fig. 6. Scalar POG

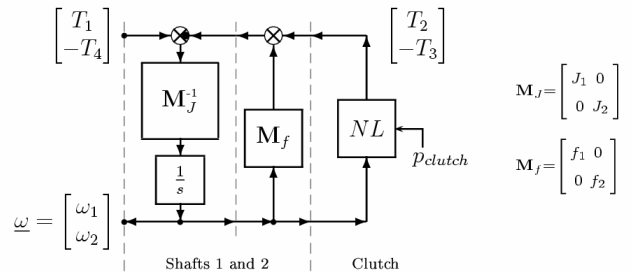


Fig. 7. Vectorial POG

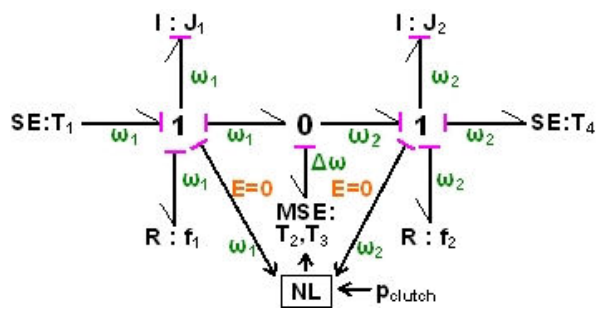


Fig. 8. Scalar Bond Graph

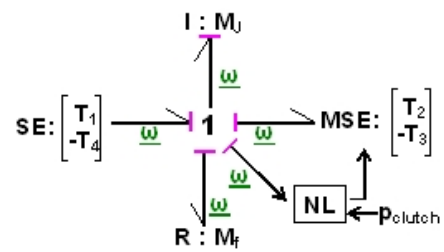


Fig. 9. Vectorial Bond Graph

EMR the product of the both associated power variables must be a power. Eight elements handle all types of energy conversion. As for the POG both a scalar and a more compact vectorial model may be generated using the same parameter definitions as given regarding the POGs (Fig. 8, Fig. 9). The measurement (real arrow) of the power variables ω_1 and ω_2 means the use of a so-called “activated bond”, i.e. “input” of zero for one power variable or one vector of power variables, i.e. a torque here, and measurement of the conjugated power variable of a Bond Graph connection. This way the power balance of the relevant node is not changed by a measurement.

The 0 node (see Appendix) equation $\Delta\omega = \omega_1 - \omega_2$ of the scalar Bond Graph shows a negative sign regarding ω_2 because the preferred power transmission direction of the bond and the transmission direction of the flow, i.e. the angular velocity ω_2 are of opposite direction. The same consideration is valid regarding the 1 nodes and torque T_s , the torque at the left side of the 0 node respectively the two friction torques. Hence the vectorial Bond Graph presents “ T_4 ” and “ T_3 ” as inputs to take into account these negative signs and the sign change at the vectorial 1 node. Thus in both cases the clutch losses are computed at the right way.

Block “NL” contains the computation of the non-linearity (relationships (4), (5), (9) and (10)) based on the input of ω_1 , ω_2 and p_{clutch} . The output must be torque T_2 and T_3 as an input for the Modulated Source Effort (MSE) by slip depending switching between the actually and the necessary clutch torque – see relationships (9) and (10).

VI. SIMULATION RESULTS

The MATLAB-Simulink™ block diagram corresponding to the vectorial POG scheme of Fig. 7 is shown in Fig. 10. Fig. 11 demonstrates the simulation of the vectorial Bond Graph according by means of an add-on block library for Simulink [20]. Because Simulink is not predestined to perform bidirectional connections the second and not visible connection of each bond is realized via the background of the software. The Simulink block diagram according to the EMR scheme of Fig. 4 is shown in Fig. 12. The commutation between both clutch models (slipping or locked) is implemented in Stateflow™ of MATLAB, which manages the Petri net (Fig. 5). The identical graphical structure may be clearly recognized for each formalism.

It has been checked that the three different block diagrams provides the same simulation with the following parameters:

$$\begin{aligned} T_1 &= 40 \text{ Nm} & T_4 &= 25 \text{ Nm} \\ J_1 &= 0.2 \text{ kg.m}^2 & J_2 &= 0.6 \text{ kg.m}^2 \\ f_1 &= 10^{-3} \text{ Nm.s} & f_2 &= 2 \cdot 10^{-3} \text{ Nm.s} \\ \mu_{stat} &= 0.28 & R_{clutch} &= 0.1 \text{ m} \\ \alpha_{dyn} &= 10^{-3} \text{ s/rad} & F_{max} &= 4000 \text{ N} \end{aligned}$$

At $t = 0$ s, the clutch is locked, the model 2 is used (Fig. 13c) and the rotation speeds are equal (Fig. 13d). The clutch release bearing is activated (Fig. 13a) at $t_1 = 0.5$ s, the maximal

clutch torque decrease (Fig. 13b). When the clutch torque is greater than the maximal clutch torque (t_2), the clutch slips. The rotation speeds are different (EMR model 1 used). The clutch release bearing is deactivated again at time t_3 . The clutch is considered to lock when both speeds become equal (t_4 , EMR model 2 used). The coefficient friction μ (Fig. 13e) is constant (μ_{stat}) when the clutch is locked and varies with the speed slip when the clutch slips according to the relationship (5). It should be noted that the transmission efficiency η (Fig. 13f) defined by (11) decreases when the slip speed increases.

$$\eta(pu) = \frac{T_4 \Omega_2}{T_1 \Omega_1} \quad (11)$$

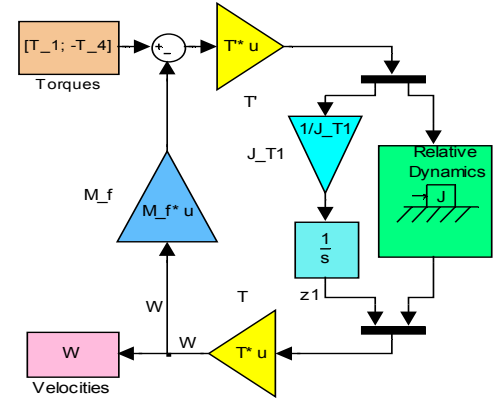


Fig. 10. Vectorial MATLAB-Simulink™ POG

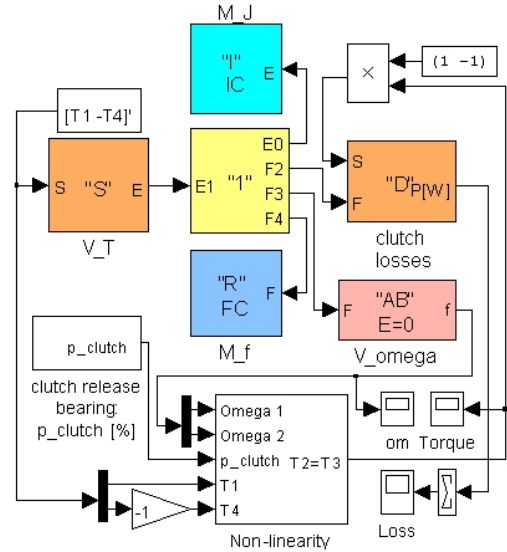


Fig. 11. Vectorial MATLAB-Simulink™ Bond Graph

VII. DISCUSSION

Three different graphical representations have been presented in order to describe the features of each tool for the modelling of systems including non-linearities.

All these descriptions are based on energetic considerations. In BG the power flows are explicitly highlighted by bonds. The power exchanged by connected elements is the

product of a flow / kinetic variable by an effort / potential variable.

In comparison with the EMR, BG and POG describe the system with the exact number of subsystems. Two 1-nodes (for the scalar models) with each a potential energy storage (I-element – sense of BG) and an energy consumer (R-element) are used. A modulated source element (sense of BG) is then defined. There is not Petri Net or something similar and no selector or something similar necessary. The non-linearity is just programmed as a control law for the modulation of the source. On the other hand, as EMR use the integral causality, the modelling is different according to the clutch state. The EMR modelling shows that a system with different states can be represented in a structured way.

For more details, details comparison between EMR, BG and POG are presented in [21].

At last, it should be noted that the use of an Hardware-In-the-Loop (HIL) simulation of the clutch has allowed to validate a control scheme, which is deduced on the inversion of the switched causal EMR [22].

VIII. CONCLUSION

A friction clutch has been modelled using different tools which are all based on energetic consideration. All models as well as customize the power flow and are proper for the use of the standard simulation software Simulink but make different demands to the user and to the modelling approach. It is shown that the modelling may be done more or less compact, with and without state switching but each case highlights essential features of the system and may be used for energy efficiency improvement. Essential differences of these modelling tools have been discussed.

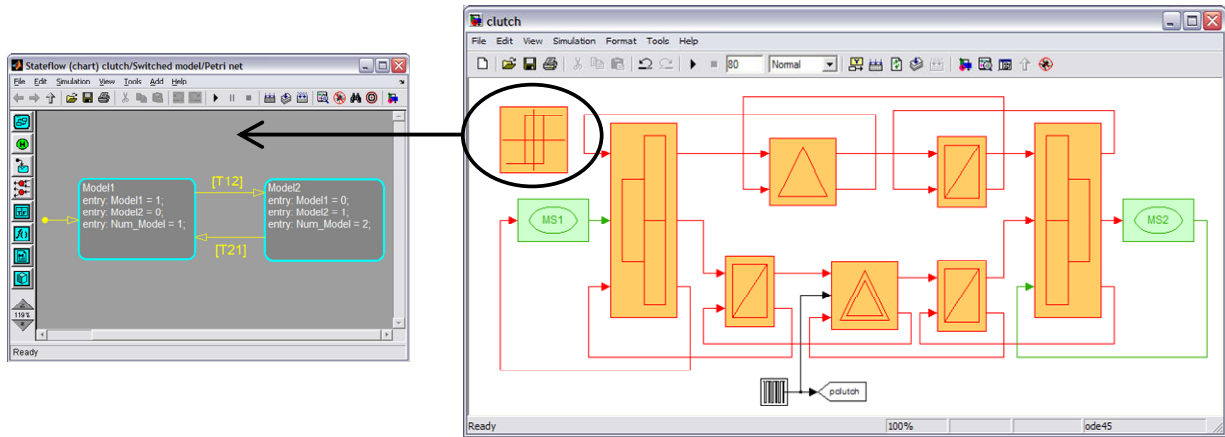


Fig. 12. MATLAB-Stateflow™ (left) and MATLAB-Simulink™ (right) EMR

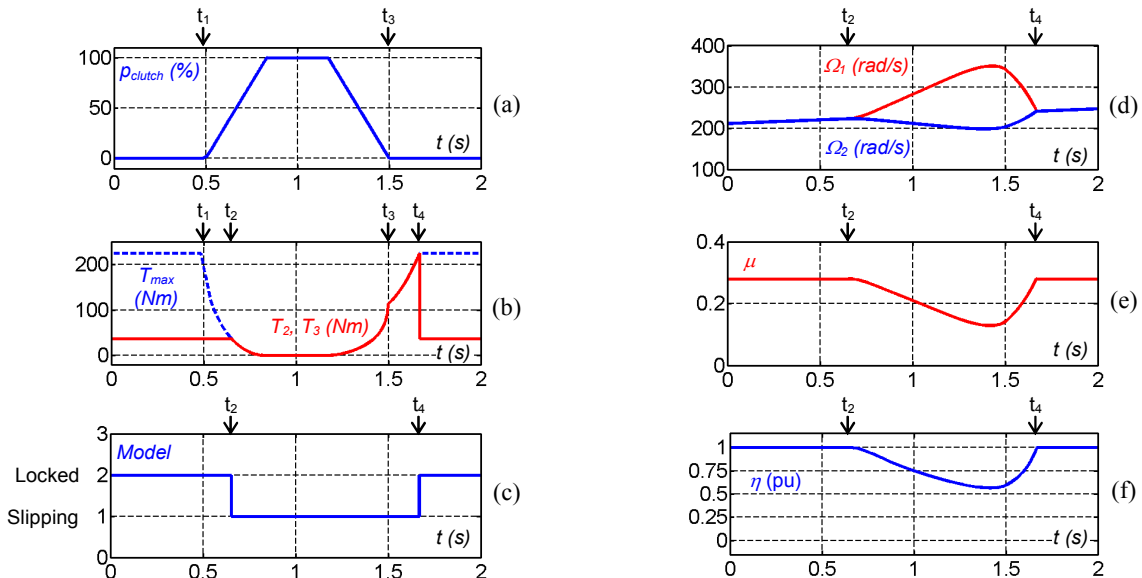


Fig. 13. Simulation results of the transmission with clutch: position of the clutch release bearing (a), clutch torque and maximal clutch torque (b), model used (c), rotation speeds (d), friction coefficient (e), and transmission efficiency (f)

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APPENDIX: SYNOPTIC OF THE GRAPHICAL DESCRIPTIONS

EMR		POG		Bond Graph	
	Mechanical source of energy		Elaboration block; $G(s)$: scalar or vectorial type; Black spot present near to an entering arrow: negative input		Energy consumer
	Element with energy accumulation		Connection block K : scalar or vectorial type		Potential energy storage
	Non-linear mechanical converter (without energy accumulation)				Node $e_1 = e_2 = \dots = e_n$ $\sum a_i f_i = 0$
	Electromechanical converter (without energy accumulation)				Node $f_1 = f_2 = \dots = f_n$ $\sum a_i e_i = 0$
	Selector of models				