DISCRETE SECOND ORDER TRAJECTORY GENERATOR WITH NONLINEAR CONSTRAINTS

**Riccardo Morselli * Roberto Zanasi *
Stefano Stramigioli ****

* *DII, University of Modena and Reggio Emilia
Via Vignolese 905, 41100 Modena, Italy
Phone: +39 59 2056161, Fax +39 59 2056126
e-mail: morselli.riccardo@unimore.it*
** *Drebbel Institute EL-CE, University of Twente
P.O.Box 217, 7500AE Enschede, The Netherlands
Phone: +31 53 4892794, Fax +31 53 4892223
e-mail: S.Stramigioli@ieee.org*

Abstract: A discrete second order trajectory generator for motion control systems is presented. The considered generator is a nonlinear system which receives as input a raw reference signal and provides as output a smooth reference signal satisfying nonlinear constraints on the output derivatives as $\mathbf{U}_M^-(\dot{x}) \leq \ddot{x} \leq \mathbf{U}_M^+(\dot{x})$ and $\mathbf{V}_M^- \leq \dot{x} \leq \mathbf{V}_M^+$. The trajectory is generated on-line and the imposed constraints can also be changed during system operation without modifying the system stability. Moreover, almost minimum time response is ensured with guaranteed no overshoot. The performances of the nonlinear generator are tested through experiments on a linear motor.

Keywords: Trajectory planning, optimal control, constraint satisfaction problems, nonlinear systems, linear motors.

1. INTRODUCTION

A common problem in many motion control applications is the tracking of standard reference signals (i.e. piecewise continuous signals such as steps, ramps, etc.) in systems that, as a consequence of energetic and constructive limitations (i.e. the torque curve of an electric motor), must satisfy bounds on the controlled output derivatives.

Linear or nonlinear regulators are commonly used to track a standard reference signal. The steady state exact tracking and the achievement of a good behaviour during transients are both on charge of the regulator. The main problem of this type of solution is that the regulator is not usually able to take into account the energetic and constructive limitations of the plant. Thus the transient response becomes a compromise between velocity and absence of overshoot.

Trajectory generators allow to separate the tracking problem from the achievement of a good behaviour during transients. When a standard reference is applied, the output of a trajectory generator tracks the reference signal satisfying on-line user-defined nonlinear bounds on the output derivatives. If this constraints on the trajectories matches the energetic and constructive limitations of the controlled plant, than the plant can follow the generated trajectory. Now a simpler regulator

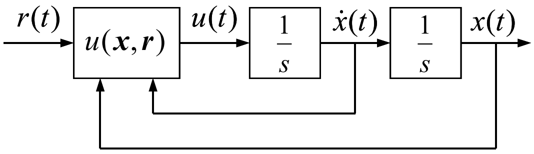

Fig. 1. Continuous time trajectory generator.

can be used to track the generated trajectory and the output derivatives are available to implement feed-forward actions. For more details on trajectory generators please refer to (Brady, 1982) and (Morselli, 2003).

This paper proposes a second order discrete time trajectory generator that satisfies nonlinear constraints on the maximum acceleration of the type: $\mathbf{U}_M^-(\dot{x}) \le \ddot{x} \le \mathbf{U}_M^+(\dot{x})$ and $\mathbf{V}_M^- \le \dot{x} \le \mathbf{V}_M^+$. This trajectory generator is the discrete time version of the generator proposed in (Zanasi and Morselli, 2001). This discretisation has been necessary for the real application on the digital control of a linear motor.

The paper is organized as follows: Sec. 2 states the control problem, Sec. 3 describes the discretization procedure that leads to the discrete time trajectory generator presented in Sec. 4. Some experimental results are presented in Sec. 5.

2. PROBLEM STATEMENT

Let's consider the chain of 2 integrators shown in Fig. 1, where $r(t)$ is the input, $\dot{r}(t)$ is the first time derivative of the input, $x(t)$ is the output, $\dot{x}(t)$, $\ddot{x}(t)$ are the first and the second time derivatives of the output and $u(t) = \ddot{x}(t)$ is the input control signal of the chain of integrators. The following assumptions hold:

1) The input reference signal $r(t)$ satisfies piecewise the condition $\ddot{r}(t) = 0$.

2) The reference input $r(t)$ and its derivative $\dot{r}(t)$ are known for $t \ge 0$.

According to this assumptions, the input reference signal is a sequence of ramps and it is continuous almost always. Square and saw tooth references belong to this category.

Let's denote with $y(t)$, $\dot{y}(t)$ the error variables:

$$y = x - r \qquad \dot{y} = \dot{x} - \dot{r}$$

Using the error variables $y(t)$ and $\dot{y}(t)$, the system shown in Fig. 1 can be described by:

$$\begin{aligned} \dot{\mathbf{y}}(t) &= \mathbf{A}_c \mathbf{y}(t) + \mathbf{b}_c\, \mathbf{u}(t) \\ \mathbf{x}(t) &= \mathbf{y}(t) + \mathbf{r}(t) \end{aligned} \tag{1}$$

where $\mathbf{x} = [x,\, \dot{x}]^T$, $\mathbf{y} = [y,\, \dot{y}]^T$, $\mathbf{r} = [r,\, \dot{r}]^T$ and

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{b}_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

In (Zanasi and Morselli, 2001) it is presented the nonlinear controller $u(\mathbf{r}, \mathbf{x})$ such that the output $x(t)$ of system (1) tracks the reference signal $r(t)$ with the following specifications:

S1) The velocity $\dot{x}$ and the acceleration u are bounded: $\mathbf{V}_M^- \le \dot{x} \le \mathbf{V}_M^+$, $|\ddot{x}| = |u| \le U_M(\dot{x})$, where $\mathbf{V}_M^+$ is the maximum velocity, $\mathbf{V}_M^-$ is the minimum velocity and $U_M(\dot{x})$ is a continuous function which is strictly positive.

S2) When a constant velocity reference $\dot{r}$ is applied (satisfying the constraints $\mathbf{V}_M^- \le \dot{r} \le \mathbf{V}_M^+$) the tracking condition $x(t) = r(t)$ is ensured in finite time t_0 and $\forall t > t_0$.

S3) When a discontinuous or a non appropriate velocity reference (not constant or not possible with the constraints $|\ddot{x}| = |u| \le U_M(\dot{x})$ or exceeding the bound values $\mathbf{V}_M^-$, $\mathbf{V}_M^+$) is applied, the perfect tracking is lost. As soon as the admissible reference is re-established, the tracking condition is achieved again in minimum time and without overshoot.

Namely the output $x(t)$ of the trajectory generator of Fig.1 is a filtered version of the raw reference signal $r(t)$. The bounds on the derivatives of $x(t)$ give the degree of smoothness of the output trajectory. The result presented in (Zanasi and Morselli, 2001) is not completely suitable for an on-line application with a digital controller for two reasons: the demanding computational effort required for the computation of the trajectories and the implementation of a continuous control law in a discrete system.

The aim of this paper is to find an approximated discrete solution of the continuous time problem expressed above. The approach will partially follow the one presented in (Zanasi *et al.*, 2000), indeed that trajectory generator is a special simpler case of the solution presented in this paper. The proposed trajectory generator can be seen in the context of the model predictive control (Rawlings, 2000), in the following a dedicated solution is presented in order to reduce the computational effort.

3. DISCRETIZATION OF THE CONTINUOUS TIME TRAJECTORY GENERATOR

By direct discretization of system (1) with sample time T, the following difference state space equation is obtained:

$$\mathbf{y}_{n+1} = \mathbf{A}_0\, \mathbf{y}_n + \mathbf{b}_0\, u_n \tag{3}$$

where $y_n = x_n - r_n$, $\dot{y}_n = \dot{x}_n - \dot{r}_n$, $\mathbf{A}_0 = e^{\mathbf{A}_c T}$, $\mathbf{b}_0 = \int_0^T e^{\mathbf{A}_c \tau} \mathbf{b}_c d\tau$ and

$$\mathbf{A}_0 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \qquad \mathbf{b}_0 = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

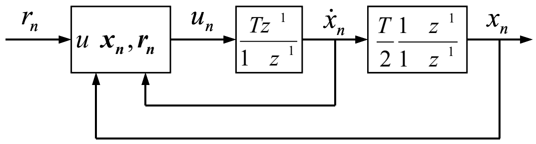

Fig. 2. Discrete time trajectory generator.

Applying to system (3) the state space transformation $\mathbf{y}_n = \mathbf{T}\,\mathbf{z}_n$, $\mathbf{z}_n = \mathbf{T}^{-1}\,\mathbf{y}_n$ where:

$$\mathbf{T} = \begin{bmatrix} T^2 & -\dfrac{T^2}{2} \\ 0 & T \end{bmatrix} \qquad \mathbf{T}^{-1} = \begin{bmatrix} \dfrac{1}{T^2} & \dfrac{1}{2T} \\ 0 & \dfrac{1}{T} \end{bmatrix}$$

one obtains the following transformed system:

$$\mathbf{z}_{n+1} = \overline{\mathbf{A}}\,\mathbf{z}_n + \overline{\mathbf{b}}\,u_n \quad \overline{\mathbf{A}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \overline{\mathbf{b}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{4}$$

System (4) is equivalent to system (3), but expressed in a normalized form with respect to the sampling period T. In the following, the two components of the state vector $\mathbf{z}_n$ will be denoted with the symbols $z_{1,n}$ and $z_{2,n}$: $\mathbf{z}_n = [z_{1,n}\ z_{2,n}]^T$.

Inverting system (4) with respect to time:

$$\mathbf{z}_{k+1} = \mathbf{A}\mathbf{z}_k - \mathbf{b}\,u_k \tag{5}$$

where

$$\mathbf{A} = \overline{\mathbf{A}}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad \mathbf{b} = \overline{\mathbf{A}}^{-1}\overline{\mathbf{b}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{6}$$

The state $\mathbf{z}_{k+1}$ precede the state $\mathbf{z}_k$ by one sampling period, namely applying the control $u_n = u_k$ the state $\mathbf{z}_n = \mathbf{z}_{k+1}$ is mapped by (4) in the state $\mathbf{z}_{n+1} = \mathbf{z}_k$.

Property 1: let $\mathbf{B}_k = [B_{1,k}\ B_{2,k}]^T$ be a point in the plane (z_1, z_2). The control u_k maps the point B_k backward into a vertical line. Indeed:

$$\mathbf{z}_{k+1} = \mathbf{A}\,\mathbf{B}_k - \mathbf{b}u_k = \begin{bmatrix} B_{1,k} - B_{2,k} \\ B_{2,k} - u_k \end{bmatrix} \tag{7}$$

Property 2: let $\mathbf{B}_{k-1}$, $\mathbf{B}_k$ and $\mathbf{B}_{k+1}$, be three points in the plane (z_1, z_2) such that $\mathbf{B}_{k+1} = \mathbf{A}\mathbf{B}_k - \mathbf{b}u_k$ and $\mathbf{B}_k = \mathbf{A}\mathbf{B}_{k-1} - \mathbf{b}u_{k-1}$, see Fig. 3. All the points belonging to the vertical strip between the two points $\mathbf{B}_k$ and $\mathbf{B}_{k+1}$ can be mapped forward into the segment between the two points $\mathbf{B}_k$ and $\mathbf{B}_{k-1}$ by a proper choice of the control input u_n.

Proof 2: let $\mathbf{B}_{k-1} = [B_{1,k-1}\ \ B_{2,k-1}]^T$, the coordinates of the two points $\mathbf{B}_k$ and $\mathbf{B}_{k+1}$ are:

$$\mathbf{B}_k = \mathbf{A}\,\mathbf{B}_{k-1} - \mathbf{b}\,u_{k-1} = \begin{bmatrix} B_{1,k-1} - B_{2,k-1} \\ B_{2,k-1} - u_{k-1} \end{bmatrix}$$

$$\mathbf{B}_{k+1} = \mathbf{A}\,\mathbf{B}_k - \mathbf{b}\,u_k = \begin{bmatrix} B_{1,k-1} - 2B_{2,k-1} + u_{k-1} \\ B_{2,k-1} - u_{k-1} - u_k \end{bmatrix}$$

A point $\mathbf{P}_n = [P_{n,1}\ \ P_{n,2}]^T$ belongings to the vertical strip between the two points $\mathbf{B}_k$ and $\mathbf{B}_{k+1}$ can be expressed as:

$$\mathbf{P}_n = \begin{bmatrix} B_{1,k} + \alpha B_{1,k+1} \\ P_{2,n} \end{bmatrix} \qquad \alpha \in [0, 1]$$

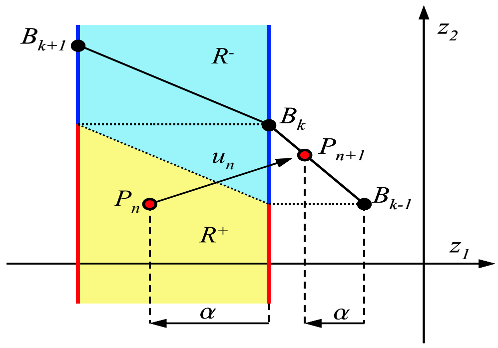

Fig. 3. Mapping properties of the space state transformation $\mathbf{T}$.

Applying forward the control input

$$u_n = B_{2,k-1} + \alpha(B_{2,k} - B_{2,k-1}) - P_{2,n} \tag{8}$$

the point $\mathbf{P}_n$ is mapped into the point $\mathbf{P}_{n+1}$:

$$\mathbf{P}_{n+1} = \mathbf{B}_{k-1} + \alpha(\mathbf{B}_k - \mathbf{B}_{k-1})$$

that is a point of the segment delimited by the two points $\mathbf{B}_k$ and $\mathbf{B}_{k-1}$. $\diamond$

Note that the control u_n depends only on the difference between the z_2 component of $\mathbf{P}_n$ and the z_2 component of $\mathbf{P}_{n+1}$. Moreover the coefficient α that identifies the horizontal position of $\mathbf{P}_n$ within the vertical strip is the same that identifies the position of $\mathbf{P}_{n+1}$ within the segment between $\mathbf{B}_k$ and $\mathbf{B}_{k-1}$, see Fig. 3. The vertical strip can be divided in two regions, R^+ and R^-, each point of R^+ (R^-) is mapped into the segment between $\mathbf{B}_k$ and $\mathbf{B}_{k-1}$ by a positive (negative) control u_n.

4. COMPUTATION OF THE DISCRETE TIME CONTROL LAW

Since $\ddot{r}(t) = 0$ almost always, the velocity reference $\dot{r}_n$ is piecewise constant, let $\dot{r}_n = \dot{r}_d$. The following relations hold:

$$\dot{y}_n = T\,z_{2,n} \qquad \dot{x}_n = \dot{r}_d + T\,z_{2,n}$$

The role of the function $U_M(\dot{x})$ is replaced by the two strictly positive functions $U^+(\dot{x}_n)$ and $U^-(\dot{x}_n)$:

$$U^+(\dot{x}_n) = U^+(\dot{r}_d + T\,z_{2,n}) > 0$$

$$U^-(\dot{x}_n) = U^-(\dot{r}_d + T\,z_{2,n}) < 0$$

Using the inverted system (5), let compute backward the following two series of points of the plane (z_1, z_2) starting from the origin $(0, 0)$:

$$\begin{aligned} \mathbf{B}^+_{k+1} &= \mathbf{A}\,\mathbf{B}^+_k - b\,u^+_k \\ \mathbf{B}^+_0 &= [0\ 0]^T \\ u^+_k &= U^+(\dot{r}_d + T\,B^+_{2,k}) \\ \mathbf{B}^-_{k+1} &= \mathbf{A}\,\mathbf{B}^-_k - b\,u^-_k \\ \mathbf{B}^-_0 &= [0\ 0]^T \\ u^-_k &= U^-(\dot{r}_d + T\,B^-_{2,k}) \end{aligned} \tag{9}$$

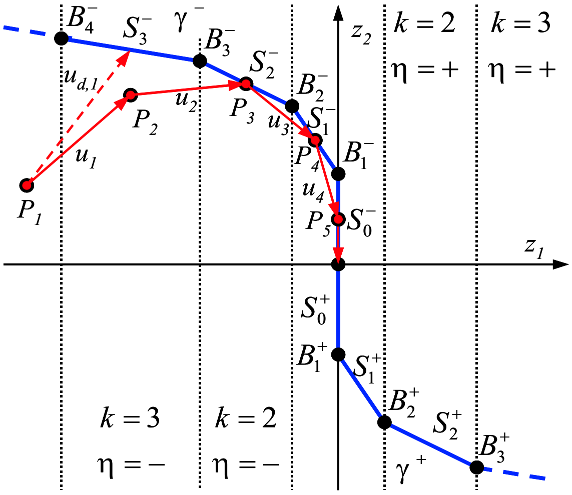

Fig. 4. Example of points B_k^η, segments $\mathbf{S}_k^\eta$ and curve γ^η.

Let η denote the symbol $+$ or $-$ such that the points defined in (9) can be briefly denoted with B_k^η. Let $\mathbf{S}_k^\eta$ denote the segment on the plane $(z_1,\, z_2)$ delimited by the two points $\mathbf{B}_k^\eta$ and $\mathbf{B}_{k+1}^\eta$. Finally, let γ^η be the set of all the segments $\mathbf{S}_k^\eta$, see Fig. 4. Given a point $\mathbf{z}_n = [z_{1,n} \quad z_{2,n}]^T$ it is possible to compute η and the integer k in the following way:

$$\eta = \begin{cases} + & \text{if} \quad z_{1,n} \geq 0 \\ - & \text{if} \quad z_{1,n} < 0 \end{cases}$$

$$k = \begin{cases} i & \text{if} \quad B_{1,i+1}^- \leq z_{1,n} < B_{1,i}^- \\ 0 & \text{if} \quad z_{1,n} = 0 \\ i & \text{if} \quad B_{1,i+1}^+ \geq z_{1,n} > B_{1,i}^+ \end{cases} \tag{10}$$

The integer k together with η identify the vertical strip, delimited by the two consecutive points $\mathbf{B}_k^\eta$ and $\mathbf{B}_{k+1}^\eta$, that contains the point $\mathbf{z}_n$. The value of k increases with the distance from the origin. From property 2 the control signal:

$$u_{d,n} = B_{2,k-1}^\eta + \alpha(B_{2,k}^\eta - B_{2,k-1}^\eta) - z_{2,n}$$

$$\alpha = \begin{cases} \dfrac{z_{1,n} - B_{1,k}^\eta}{B_{1,k+1}^\eta - B_{1,k}^\eta} & \text{if} \quad k \neq 0 \\ 0 & \text{if} \quad k = 0 \end{cases} \tag{11}$$

maps the point $\mathbf{z}_n$ belongings to the strip k into the segment $\mathbf{S}_{k-1}^\eta$ or into the origin when $k = 0$.

As shown later, for any point $\mathbf{z}_n = [z_{1,n} \quad z_{2,n}]^T$ the corresponding control u_n of the generated trajectory will be bounded between the two values $u_n^{\max}$ and $u_n^{\min}$:

$$u_n^{\max} = \begin{cases} u_{d,n} & \text{if} \quad u_{d,n} \leq \max(u_k^+, u_{k-1}^+) \\ U^+(\dot{r}_d + T\, z_{2,n}) & \text{else} \end{cases}$$

$$u_n^{\min} = \begin{cases} u_{d,n} & \text{if} \quad u_{d,n} \geq \min(u_k^-, u_{k-1}^-) \\ U^-(\dot{r}_d + T\, z_{2,n}) & \text{else} \end{cases} \tag{12}$$

where $u_k^\eta = U^\eta(\dot{r}_d + T\, B_{2,k}^\eta)$ and k is the index of the vertical strip that contains the point $\mathbf{z}_n$.

If the sampling time T is sufficiently small and if the two functions $U^+(\dot{x}_n)$ and $U^-(\dot{x}_n)$ are sufficiently smooth, then $u_n^{\max} \simeq U^+(\dot{x}_n)$ and $u_n^{\min} \simeq U^-(\dot{x}_n)$. Therefore the two functions $U^\pm(\dot{x}_n)$ give a good approximation of the acceleration constraints that the generated trajectory satisfies.

Proposition: the time optimal trajectory to the origin with acceleration u_n constrained between $u_n^{\max}$ and $u_n^{\min}$ is obtained saturating the control $u_{d,n}$ between $u_n^{\max}$ and $u_n^{\min}$:

$$u_n = \max(u_n^{\min}, \min(u_n^{\max}, u_{d,n})) \tag{13}$$

Proof. Thanks to the property 2, the control signal $u_{d,n}$ given in (11) maps the point $\mathbf{z}_n = [z_{1,n} \quad z_{2,n}]^T$ belonging to the strip k to the segment $\mathbf{S}_{k-1}^\eta$ (i.e. the control $u_{d,1}$ in Fig. 4), then from $\mathbf{S}_{k-1}^\eta$ to $\mathbf{S}_{k-2}^\eta$ and so on until the origin. The three possible cases are: 1) $\mathbf{z}_n \in \mathbf{S}_k^\eta$, 2) $\mathbf{z}_n$ is "close" to $\mathbf{S}_k^\eta$ and 3) $\mathbf{z}_n$ is "far" from $\mathbf{S}_k^\eta$.

1) If the point $\mathbf{z}_n$ belongs to the segment $\mathbf{S}_k^\eta$ (i.e. the point P_3 in Fig. 4) the control signal u_n defined in (13) is equal to $u_{d,n}$ and therefore u_n inherits the mapping properties of $u_{d,n}$ (i.e. the generated trajectory follows the segments $\mathbf{S}_k^\eta$, $\mathbf{S}_{k-1}^\eta$,... until the origin). Indeed for the points on the segment $\mathbf{S}_k^\eta$ the control $u_{d,n}$ satisfies:

$$\begin{aligned} u_{d,n} &= B_{2,k-1}^\eta + \alpha(B_{2,k}^\eta - B_{2,k-1}^\eta) - z_{2,n} \\ &= B_{2,k-1}^\eta + \alpha(B_{2,k}^\eta - B_{2,k-1}^\eta) + \ldots \\ &\quad - B_{2,k}^\eta - \alpha(B_{2,k+1}^\eta - B_{2,k}^\eta) \\ &= \alpha(B_{2,k}^\eta - B_{2,k+1}^\eta) + (1-\alpha)(B_{2,k-1}^\eta - B_{2,k}^\eta) \\ &= \alpha\, u_k^\eta + (1-\alpha)\, u_{k-1}^\eta \in [u_n^{\min}, u_n^{\max}] \end{aligned}$$

2) If the point $\mathbf{z}_n$ belongs to the strip k but not to the segment $\mathbf{S}_k^\eta$, the control $u_{d,n}$ tends to bring the trajectory to the point $\mathbf{z}_{n+1}$ on the segment $\mathbf{S}_{k-1}^\eta$. If $u_{d,n} \in [u_n^{\min}, u_n^{\max}]$ then $u_n = u_{d,n}$ and the trajectory is actually mapped to the segment $\mathbf{S}_{k-1}^\eta$ and then (case 1) from a segment to the next until the origin (i.e. the point P_2 in Fig. 4).

3) If $u_{d,n} \notin [u_n^{\min}, u_n^{\max}]$ the control u_n is limited within $u_n \in [U^+(\dot{r}_d + T\, z_{2,n}), U^-(\dot{r}_d + T\, z_{2,n})]$. The point $\mathbf{z}_{n+1}$ will not reach any segment $\mathbf{S}_k^\eta$ but it will be "closer" to the curve γ^η (i.e. the point P_1 in Fig. 4). Indeed note that the control $u_{d,n}$ is positive (negative) for all the points below (above) the curve γ^η and not belongings to case 2 (see Fig. 3). Since $\text{sgn}(u_n) = \text{sgn}(u_{d,n})$ the components $\mathbf{z}_{2,n+1}$, $\mathbf{z}_{2,n+2}$,... will increase (decrease). Therefore, sooner or later the trajectory will be close enough to the curve γ^η to allow the mapping on a segment $\mathbf{S}_k^\eta$ (case 2).

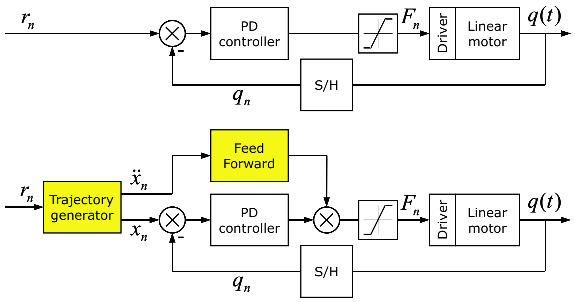

Fig. 5. Control schemes with(bottom) and without(top) trajectory generator

The time optimality is a consequence of the backward integration and optimal control theory, see (Lee and Markus, 1967). ⋄

Control law The control signal u_n of the system (3) given by equations (13), (12), (11) and (10), generates the minimum time trajectory that tracks the reference signal with acceleration bounded between $u_n^{\max}$ and $u_n^{\min}$.

The velocity bounds $\mathbf{V}_M^- \leq \dot{x} \leq \mathbf{V}_M^+$ can be easily added with a sight modification of the control law, as described in (Morselli and Zanasi, 2002). Let

$$u_n^{V+} = \frac{1}{T}(V_M^+ - \dot{x}_n) \qquad u_n^{V-} = \frac{1}{T}(V_M^- - \dot{x}_n)$$

u_n^{V+} and u_n^{V-} represent the control signal that bring the system to the maximum or to the minimum allowed speed respectively. The law:

$$u_n = \max(u_n^{\min}, u_n^{V-}, \min(u_n^{\max}, u_n^{V+}, u_{d,n})) \quad (14)$$

generates the minimum time trajectory that tracks the reference with acceleration bounded between $u_n^{\max}$ and $u_n^{\min}$ and speed bounded between V^+ and V^-.

The time behaviours of the variables $\mathbf{r}_n$, $\mathbf{x}_n$, u_n and $U^{\pm}(\dot{x}_n)$ obtained with this control law are shown in Fig. 6. These trajectories are obtained with the constraints (15) described in the next section. Two ramps are given as reference signal, after a finite time the output x_n of the trajectory generator tracks exactly the reference signal r_n. As shown in Fig. 6 (bottom) the tracking condition is achieved keeping the control input u_n bounded between $U^{\pm}(\dot{x}_n)$.

The main computational effort of the control law is for the research of the index k. In some simple cases it is possible to find an explicit relation to compute k as in (Zanasi *et al.*, 2000). Unfortunately an explicit relation is usually not easy to be found. If the two functions $U^{\eta}(\dot{x})$ are simple enough, it is possible to compute the points $\mathbf{B}_k^{\eta}$ explicitly and this reduces the computational effort to find k. The research of the index k is much simpler and less demanding than the continuous integration required in (Zanasi and Morselli, 2001). The research of the index k can be seen as a simplified discrete integration. The proposed algorithm is now implementable in a discrete time control system, the following section shows some experimental results.

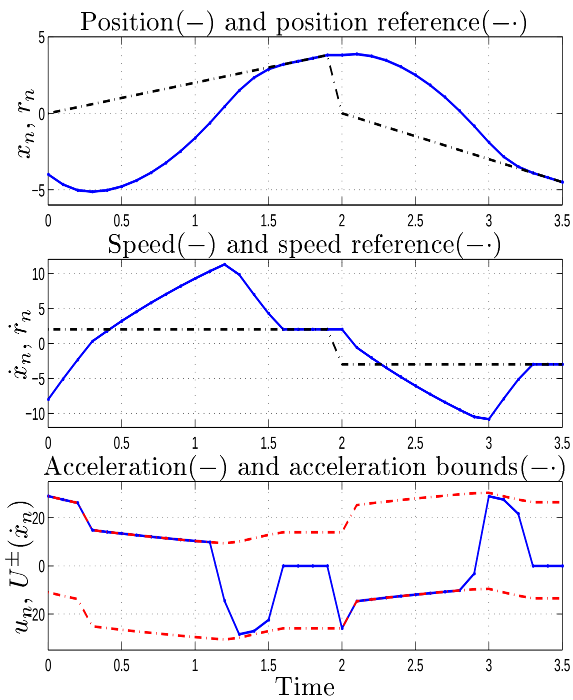

Fig. 6. Example of reference tracking. Solid lines: control u_n, speed $\dot{x}_n$ and position x_n. Dash-dot lines: reference signals $\dot{r}_n$, r_n and acceleration constraints $U^{\pm}(\dot{x}_n)$.

5. EXPERIMENTAL RESULTS

The experimental set-up is composed by a linear motor (Philips LIMMS) with moving mass of 5Kg, stroke length 600mm, maximum absolute force limited below 80N. The translator is affected by a Coulomb friction whose amplitude is not constant along the stroke, moreover the back electromotive force limits the acceleration at high translator speeds. Hence the maximum acceleration of the translator is limited by:

$$U^+(\dot{x}) = \begin{cases} U - U_0\,\mathrm{sgn}(\dot{x}) - U_1\dot{x} & \text{if} \quad \dot{x} \neq 0 \\ U - U_0 & \text{if} \quad \dot{x} = 0 \end{cases} \quad (15)$$
$$U^-(\dot{x}) = -U^+(\dot{x})$$

where the coefficients are: $U = 12.5\ \mathrm{m}/s^2$, $U_0 = 2.5\ \mathrm{m}/s^2$ and $U_1 = 4\ s^{-1}$. This choice is conservative in order to obtain a trajectory that can be followed by the translator taking into account the variable amplitude of the Coulomb friction and other parameters uncertainty.

The performances of the two control schemes shown in Fig. 5 are compared. The input position reference is a square signal with amplitude between 150mm and 450mm and a period of 8 seconds. The PD controller is the same in both schemes. The proportional coefficient was chosen

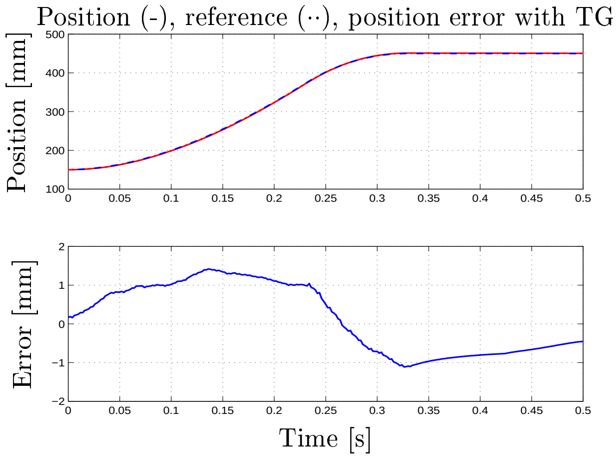

Fig. 7. Step response. Top: position (solid) and generated trajectory (dashed). Bottom: tracking error.

to fit a requirement on the steady state error, the derivative coefficient ensures the fastest settling time without overshoot. The trajectory is generated on-line.

As shown in Fig. 7, the linear motor is able to follow the generated trajectory. The knowledge of the physical limits of the motor is embedded in the parameters of the trajectory generator, thus the settling time is lower when the trajectory generator is used, see Fig. 8 where the vertical dashed lines denote the instant when the position reaches the final position ±1mm.

The lower settling time is obtained without exceeding the constraints on the maximum force, see Fig. 9. Until 0.25s the response of the control system without trajectory generator is faster, this is due to the need of choosing the trajectory parameters in a conservative way. A trajectory generator able to improve the first part of the response is under investigation.

6. CONCLUSIONS

A discrete second order trajectory generator for motion control systems was presented and tested. The possibility to choose the parameters of the generated trajectory allows to shape the motion of the translator taking into account any requirements of the type $\mathbf{U}_M^-(\dot{x}) \leq \ddot{x} \leq \mathbf{U}_M^+(\dot{x})$ and $\mathbf{V}_M^- \leq \dot{x} \leq \mathbf{V}_M^+$. Hence the generated trajectory can be chosen in order to optimize either the settling time or other features like the acceleration, i.e. to achieve smoother starts and stops.

ACKNOWLEDGEMENTS

The authors wish to thank the Control Engineering Department of the University of Twente (The Netherlands) and Dr. Mark Verwoerd for his valuable collaboration.

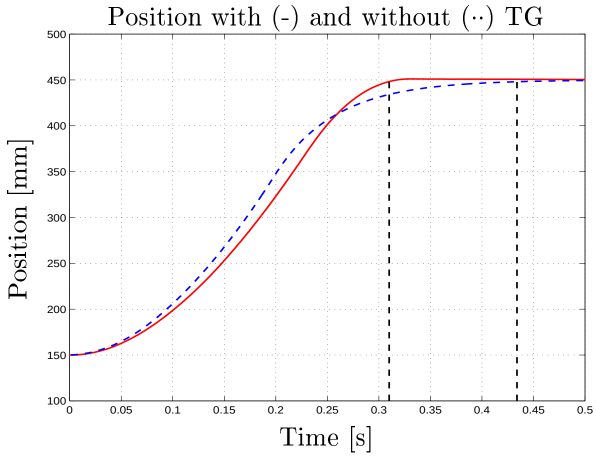

Fig. 8. Step response: position with (solid) and without (dashed) trajectory generator.

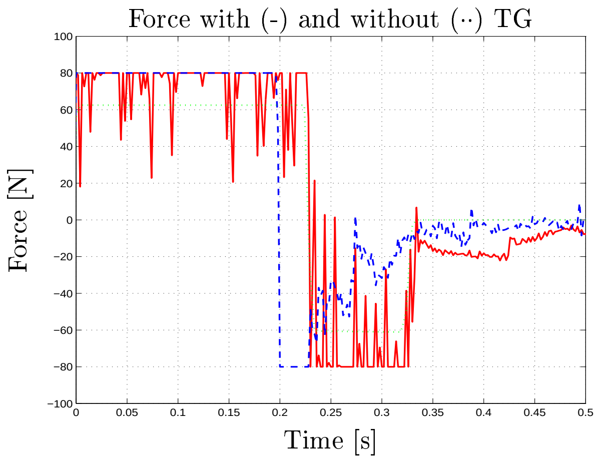

Fig. 9. Step response: force reference with (solid) and without (dashed) trajectory generator.

REFERENCES

Lee E.B., L. Markus (1967). *Foundations of optimal control theory*, John Wiley.

Brady M. (1982). Trajectory planning. In *Robot Motion: Planning and Control*, M. Brady et al., MIT Press, Cambridge, MA, pp. 221-243.

Rawlings J.B. (2000). Tutorial Overview of Model Predictive Control. *IEEE Control Systems Magazine*, Vol. 20, pp. 38-52.

Zanasi R., Guarino Lo Bianco C., Tonielli A. (2000). Nonlinear Filters for the Generation of Smooth Trajectories, *Automatica*, no. 36, pp. 439-448.

Zanasi R. and Morselli R. (2001). Second Order Smooth Trajectory Generator with Nonlinear Constraints. *ECC'01*, Porto, Portugal.

Morselli R. and Zanasi R. (2002). Third Order Trajectory Generator Satisfying Velocity, Acceleration and Jerk Constraints. *Proceedings of IEEE Conference on Control Applications CCA'02*, Glasgow, Scotland.

Morselli R. (2003). Nonlinear Trajectory Generators for Motion Control Systems. *Ph.D. Thesis, University of Modena and Reggio Emilia, Italy.*